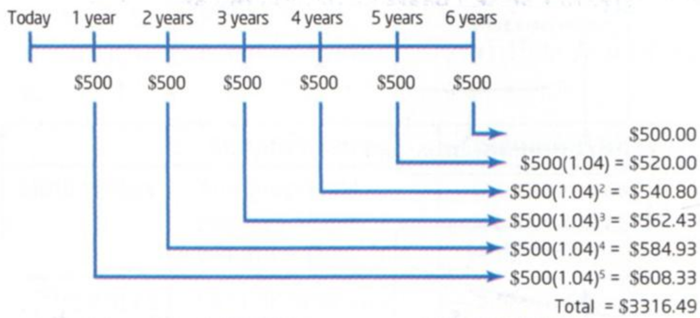
Assignment 8

1. $500 is invested annually for 6 years into a fund that pays 4% per year, compounded annually.

a) Draw a time line representing the future value of each of the following ordinary annuities.



b) Determine the variables PMT, *i*, and *n* for the annuity.

PMT = 500

$i = 0.04$

$n = 6$

c) Calculate the future value of the annuity by plugging in the values from part b into the annuity formula.

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

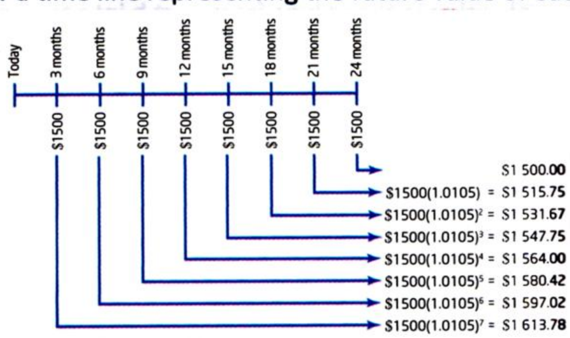
$$= 500\left[\frac{(1+0.04)^6 - 1}{0.04}\right]$$

$$= 3316.49$$

∴ The future value of the annuity is $3316.49

2. $1500 is invested quarterly for 2 years into a fund that pays 4.2% per year, compounded quarterly.

a) Draw a time line representing the future value of each of the following ordinary annuities.



b) Determine the variables PMT, *i*, and *n* for the annuity.

PMT = 1500

$i = \frac{0.042}{4} = 0.0105$

$n = 4 \frac{\text{times}}{\text{year}} \times 2\text{ years} = 8\text{ times}$

c) Calculate the future value of the annuity by plugging in the values from part b into the annuity formula.

$$FV = 1500\left[\frac{(1+0.0105)^8 - 1}{0.0105}\right]$$

$$= 12\,450.38$$

∴ The future value of the annuity is $12 450.38

3. Bronwyn is in grade 11 and has worked at a restaurant for just over a year. At the end of each month, she deposited $400 into an account that paid 3% interest per year, compounded monthly.

a) How much is in Bronwyn's account after 1 year?

PMT = 400

$i = \frac{0.03}{12} = 0.0025$

$n = 1\text{ year} \times \frac{12\text{ times}}{\text{year}} = 12\text{ times}$

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 400\left[\frac{(1+0.0025)^{12} - 1}{0.0025}\right]$$

$$= 4\,866.55$$

∴ Bronwyn's account will have $4866.55 after 1 year

b) If she continues, will the amount in her account after 2 years be double the answer to part a)?

Explain without using a calculator using what you know about annuities

No, it will be more than double because her savings from the first year will be accumulating interest.

c) Now calculate the amount that will be in Bronwyn's account after 2 years.

PMT = 400

$i = \frac{0.03}{12} = 0.0025$

$n = 2\text{ years} \times \frac{12\text{ times}}{\text{year}} = 24\text{ times}$

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 400\left[\frac{(1+0.0025)^{24} - 1}{0.0025}\right]$$

$$= 9881.13$$

∴ Bronwyn's account will have $9881.13 after 2 years.

4. Quarterly payments of $750 are made for 2 years at 9% annual interest, compounded quarterly.

a) Determine the variables PMT, *i*, and *n* for the annuity.

PMT = 750

$i = \frac{0.09}{4} = 0.0225$

$n = 2\text{ years} \times \frac{4\text{ times}}{\text{year}} = 8\text{ times}$

b) Calculate the future value of the annuity by plugging in the values from part b into the annuity formula.

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 750\left[\frac{(1+0.0225)^8 - 1}{0.0225}\right]$$

$$= 6\,494.37$$

∴ the future value of the annuity is $6 494.37

5. Determine the lump-sum amount needed to generate a retirement income of $30 000 per year for 20 years, assuming interest at 5% per year, compounded annually.

PMT = 30 000

$i = \frac{0.05}{1} = 0.05$

$n = 20\text{ years} \times \frac{1\text{ time}}{\text{year}} = 20\text{ times}$

$$PV = PMT\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$= 30\,000\left[\frac{1 - (1+0.05)^{-20}}{0.05}\right]$$

$$= 373\,866.31$$

∴ The lump sum amount needed to generate a retirement income of $30 000 per year is $373 866.31

6. a) Rearrange the present value of an annuity formula to solve for the payment. [T: 3 marks]

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$\frac{PV}{\left[\frac{1-(1+i)^{-n}}{i}\right]} = PMT$$

$$PMT = \frac{PV \times i}{1-(1+i)^{-n}}$$

b) Beth has a \$4500 loan payable over 12 months at 9% annual interest, compounded monthly. Use the formula from part a) to determine the monthly payment due at the end of each month.

$PV = 4500$ $i = \frac{0.09}{12} = 0.0075$ $n = 12\text{ months} \times \frac{1\text{ time}}{\text{month}} = 12\text{ times}$

$$PMT = \frac{4500 \times 0.0075}{1-(1+0.0075)^{-12}}$$

$$= \frac{33.75}{1-(1.0075)^{-12}}$$

$$= 393.53$$

c) What is the total amount that Beth will repay?

Beth makes monthly payments of \$393.53 for 12 months

So Beth pays

\$393.53 × 12 = \$4722.38.

∴ Beth's total amount that she repays is \$4722.38

7. Determine the value of PV, *i*, and *n* in each of the following situations.

a) \$5000 is to be repaid quarterly for 3 years with interest at 8.5% per year, compounded quarterly

$PV = 5000$

$i = \frac{0.085}{4} = 0.02125$

$n = 3\text{ years} \times \frac{4\text{ times}}{\text{year}} = 12\text{ times}$

b) a \$15 000 car loan at 7% annual interest, compounded monthly, is to be repaid with monthly payments for 4 years

$PV = 15\ 000$

$i = \frac{0.07}{12} = 0.00583\overline{333}$

$n = 4\text{ years} \times \frac{12\text{ times}}{\text{year}} = 48\text{ times}$

c) a \$22 000 business start-up loan is to be repaid with annual payments for 5 years, and the interest rate on the loan is 6.5% per year, compounded annually

$PV = 22\ 000$

$i = \frac{0.065}{1} = 0.065$

$n = 5\text{ years} \times \frac{1\text{ time}}{\text{year}} = 5\text{ times}$

d) \$2750 was borrowed to purchase a new drum set, and the loan is to be repaid in monthly instalments over 2 years, with interest at 8% per year, compounded monthly

$PV = 2750$

$i = \frac{0.08}{12} = 0.06\overline{666}$

$n = 2\text{ years} \times \frac{12\text{ times}}{\text{year}} = 24\text{ times}$

8. a) Rearrange the future value of an ordinary simple annuity formula

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

to solve for the payment.

$$\frac{FV}{\left[\frac{(1+i)^n-1}{i}\right]} = PMT$$

so $$PMT = \frac{FV \times i}{(1+i)^n - 1}$$

b) Use the formula from part a) to calculate the annual payment needed to generate \$3000 in 4 years if interest is earned at 5% per year, compounded annually.

$FV = 3000$ $\quad i = \frac{0.05}{1} = 0.05$

$n = 4\text{ years} \times \frac{1\text{ time}}{\text{year}} = 4\text{ times}$

$$PMT = \frac{3000 \times 0.05}{(1+0.05)^4 - 1}$$

$$= \frac{150}{(1.05)^4 - 1}$$

$$= 696.04$$

∴ Payments of \$696.04 must be made.

9. Calculate the total amount paid for the duration of each loan.

a) \$247 per month for 48 months

$$A = \frac{\$247}{\text{month}} \times 48\text{ months}$$

$$= \$11\,856$$

b) quarterly payments of \$498.50 for 3 years

$$A = \left(\frac{\$498.50}{\text{time}} \times \frac{4\text{ times}}{\text{year}}\right) \times 3\text{ years}$$

$$= \frac{\$1994}{\text{year}} \times 3\text{ years}$$

$$= \$5982$$

c) \$366 per month for 5 years

$$A = \frac{\$366}{\text{month}} \times 5\text{ years}$$

$$= \frac{\$366}{\cancel{\text{month}}} \times 5\cancel{\text{years}} \times \frac{12\text{ months}}{\cancel{\text{year}}}$$

$$= \$21\,960$$

d) weekly payments of \$212.47 for 25 years

$$A = \frac{\$212.47}{\text{week}} \times 25\text{ years}$$

$$= \frac{\$212.47}{\cancel{\text{week}}} \times 25\cancel{\text{years}} \times \frac{52\text{ weeks}}{\cancel{\text{year}}}$$

$$= \$276\,211$$

10. Determine the monthly payment and the total interest paid on $3000 borrowed at 8% annual interest, compounded monthly, for

a) 1 year

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 1\text{ year} \times \frac{12\text{ times}}{\text{year}} = 12\text{ times}$

$PMT = \frac{PV \times i}{1-(1+i)^{-n}}$

$= \frac{3000 \times 0.00666}{1-(1+0.00666)^{-12}}$

$= 260.97$

b) 2 years

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 2\text{ years} \times \frac{12\text{ times}}{\text{year}} = 24\text{ times}$

$PMT = \frac{PV \times i}{1-(1+i)^{-n}}$

$= \frac{3000 \times 0.00\overline{666}}{1-(1+0.00666)^{-24}}$

$= 135.68$

c) 3 years

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 3\text{ years} \times \frac{12\text{ times}}{\text{year}} = 36\text{ times}$

$PMT = \frac{3000 \times 0.00666}{1-(1+0.00666)^{-36}}$

$= 94.01$

d) 4 years

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 4\text{ years} \times \frac{12\text{ times}}{\text{year}} = 48\text{ times}$

$PMT = \frac{3000 \times 0.00666}{1-(1+0.00666)^{-48}}$

$= 73.24$

e) 5 years

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 5\text{ years} \times \frac{12\text{ times}}{\text{year}} = 60\text{ times}$

$PMT = \frac{3000 \times 0.00\overline{666}}{1-(1+0.00666)^{-60}}$

$= 60.83$

f) 7 years

$PV = 3000$

$i = \frac{0.08}{12} = 0.00\overline{666}$

$n = 7\text{ years} \times \frac{12\text{ times}}{\text{year}} = 84\text{ times}$

$PMT = \frac{3000 \times 0.006666}{1-(1+0.00666)^{-84}}$

$= 46.76$

11. Bethany Ward is an athlete who believes that her playing career will last 7 years. To prepare for her future, she deposits $22,000 at the end of each year for 7 years in an account paying 6% compounded annually. How much will she have on deposit after 7 years?

$PMT = \$22\,000$ $n = 7 \text{ years} \times \frac{1 \text{ time}}{\text{year}} = 7 \text{ times}$

$i = \frac{0.06}{1} = 0.06$

$$FV = 22\,000\left[\frac{(1+0.06)^7 - 1}{0.06}\right]$$

$$= 184\,664.43$$

∴ Bethany Ward will have \$184 664.43 after 7 years.

12. John Cross and Wendy Mears are both graduates of the Brisbane Institute of Technology (BIT). They both agree to contribute to the endowment fund of BIT. John says that he will give $500 at the end of each year for 9 years. Wendy prefers to give a lump sum today. What lump sum can she give that will equal the present value of John's annual gifts, if the endowment fund earns 7.5% compounded annually?

John

$PMT = 500$

$i = \frac{0.075}{1} = 0.075$

$n = 9 \text{ years} \times \frac{1 \text{ time}}{\text{year}} = 9 \text{ times}$

$$PV = 500\left[\frac{1-(1+0.075)^{-9}}{0.075}\right]$$

$$PV = 3189.44$$

∴ Wendy would have to donate \$3189.44 today to match John's

13. A car costs $19,000. After a down payment of $2000, the balance will be paid off in 36 equal monthly payments with interest of 6% per year on the unpaid balance. Find the amount of each payment.

$PV = 19000 - 2000$
$= 17000$

$n = 36$

$i = \frac{0.06}{12} = 0.005$

$$PV = PMT\left[\frac{1-(1+i)^{-n}}{i}\right]$$

rearrange for PMT that you did for #6a)

$$PMT = \frac{PV \times i}{1-(1+i)^{-n}} = \frac{17000 \times 0.005}{1-(1+0.005)^{-36}}$$

$$= 517.17$$

∴ A monthly payment of \$517.17 will be needed.

14. Choose a question from #1-13 that you are confident in answering correctly. You will teach how to solve this question in front of the entire class. [C: 5 marks]