

Solutions

$K \overline{25} T$

MCF 3M - Mr. Choi

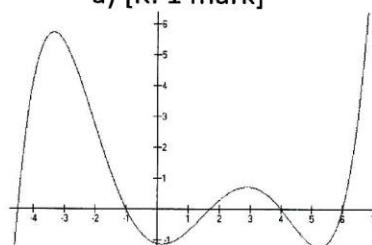
mr-choi.weebly.com

Due: July 15, 2013

Assignment 5

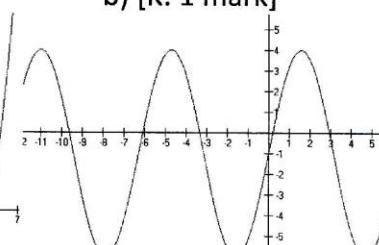
1. Determine whether or not each graph is periodic.

a) [K: 1 mark]



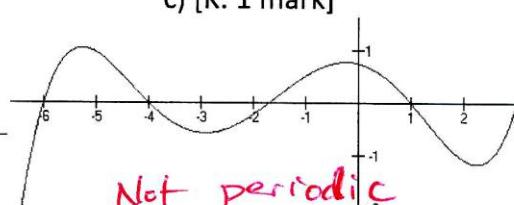
Not periodic

b) [K: 1 mark]



Periodic

c) [K: 1 mark]



Not periodic

2. Ocean tides rise and fall at sites around the world. The table shows the depth of water at one location for a 2-day period.

Time	Depth (m)
3:42	3.16 low tide
9:48	12.13 high tide
16:14	2.85 low tide
22:17	12.01 high tide
4:38	2.90 low tide
10:40	12.32 high tide
17:04	2.61 low tide
23:05	12.32 high tide

1 period

- a) Sketch a time graph of the tide depths.

[C: 4 marks]

- b) Estimate the period and the amplitude of the tide cycle. [A: 2 marks]

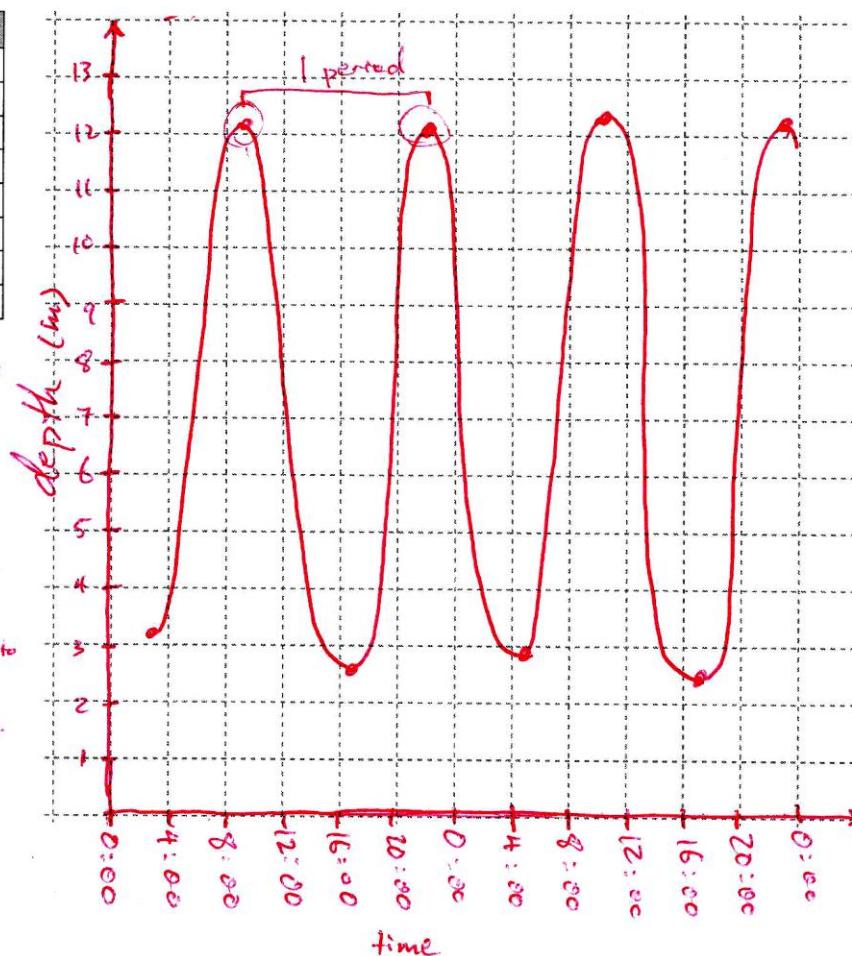
1 Period runs from 9:48 to 22:17, so one period is approximately 12.5 hours.

∴ the period is 12.5 hours.

- c) Estimate the mean depth of the water [A: 2 marks]

Mean depth is in the middle between the max and min.

so, find amplitude first and then subtract it from max to get you the mean (middle).



$$a = \frac{\max - \min}{2}$$

$$= \frac{12.13 - 2.85}{2}$$

= 4.64 is your amplitude.

$$\text{Mean} = \max - a$$

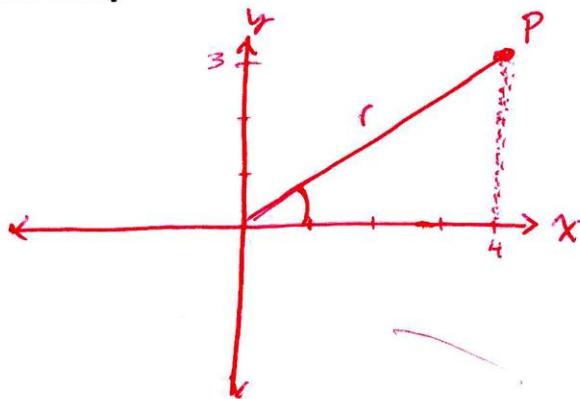
$$= 12.13 - 4.64$$

$$= 7.49 \text{ m is your mean depth.}$$

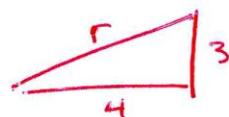
3. An angle is in standard position with its terminal point, P, given.

a) P(4, 3)

- i) Sketch the angle in standard position
[C: 2 marks]



- ii) Find the radius of the circle in exact form
[K: 2 marks]



$$c^2 = a^2 + b^2$$

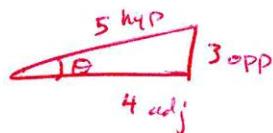
$$r^2 = 3^2 + 4^2$$

$$\sqrt{r^2} = \sqrt{9+16}$$

$$r = \sqrt{25}$$

$$r = 5$$

- iii) Find the angle to the nearest tenth of a degree. [K: 2 marks]



YOUR CHOICE

$$\sin^{-1}\left(\frac{3}{5}\right) = 36.9^\circ$$

$$\theta = 36.9^\circ$$

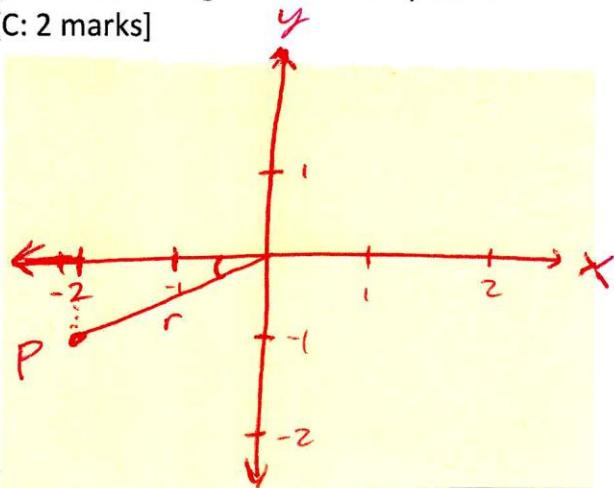
you can use
 ~~$\sin^{-1}\left(\frac{\theta}{r}\right) = \theta$~~

$$\text{or } \cos^{-1}\left(\frac{4}{5}\right) = \theta$$

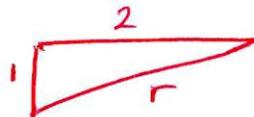
$$\text{or } \tan^{-1}\left(\frac{3}{4}\right) = \theta$$

b) P(-2, -1)

- i) Sketch the angle in standard position
[C: 2 marks]



- ii) Find the radius of the circle in exact form
[K: 2 marks]



$$c^2 = a^2 + b^2$$

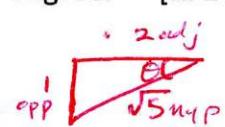
$$r^2 = 1^2 + 2^2$$

$$r^2 = 1 + 4$$

$$\sqrt{r^2} = \sqrt{5}$$

$$r = \sqrt{5}$$

- iii) Find the angle to the nearest tenth of a degree. [K: 2 marks]



you can use
 ~~$\sin^{-1}\left(\frac{\theta}{r}\right) = \theta$~~

$$\sin^{-1}\left(\frac{2}{\sqrt{5}}\right) = \theta$$

$$\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = \theta$$

$$\tan^{-1}\left(\frac{2}{1}\right) = \theta$$

YOUR CHOICE

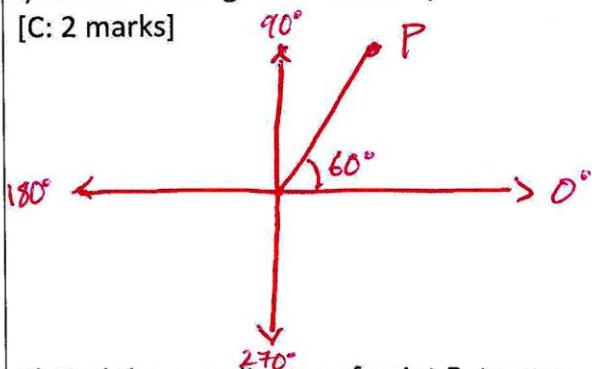
$$\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) = 26.6^\circ$$

$$\theta = 26.6^\circ$$

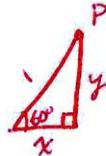
4. A terminal point, P(x, y), on the unit circle forms the given angle in standard position.

a) 60°

- i) Sketch the angle in standard position
[C: 2 marks]



- ii) Find the coordinates of point P, to one decimal place
[K: 2 marks]



$$\cos(60^\circ) = \frac{x}{1}$$

$$\text{so } x = 0.5$$

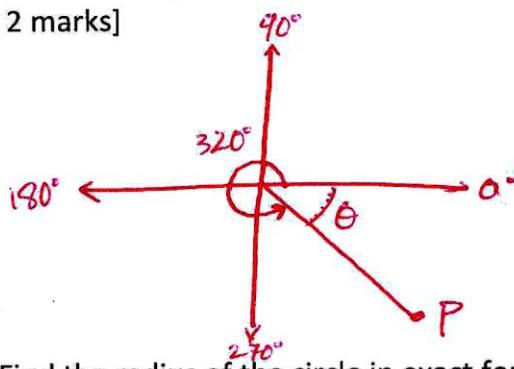
$$\sin(60^\circ) = \frac{y}{1}$$

$$\text{so } y = 0.9$$

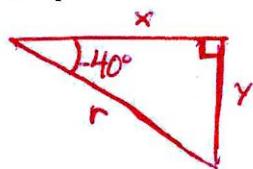
$\therefore P$ is at $(0.5, 0.9)$

b) 320°

- i) Sketch the angle in standard position
[C: 2 marks]



- ii) Find the radius of the circle in exact form
[K: 2 marks]

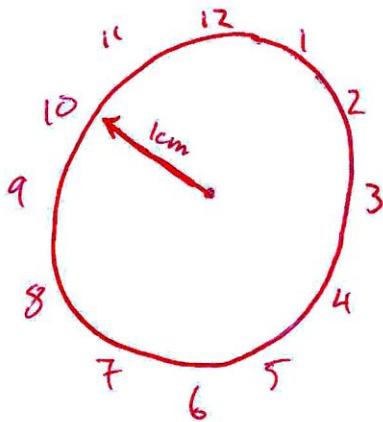


$$\tan(40^\circ) = \frac{y}{r}$$

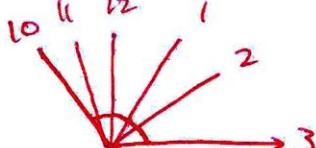
$$= -\frac{0.84}{1} \quad (1, -0.84)$$

5. The minute hand of a watch is 1 cm in length and is pointing at the number 10 on the watch face.

- a) Draw a diagram
[C: 2 marks]



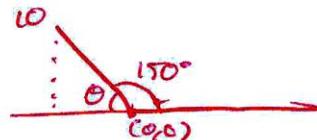
- b) What angle does it make with the number 3?
[A: 2 marks]



Since there's 360° in a circle and 12 hours on a clock, each hour is 30° .

With 5 hours separating 10 and 3, there is an angle of 150°

- c) If the centre of the watch face is plotted as $(0, 0)$ on a graph, what ordered pair would represent the number 10 on the watch face?
[T: 2 marks]

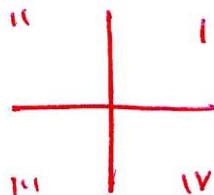


$$\theta = 180^\circ - 150^\circ = 30^\circ$$

Remember Special triangle with 30°

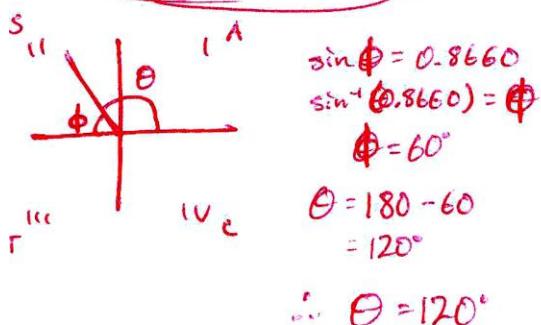


So 10 would be at $(-\sqrt{3}, 1)$

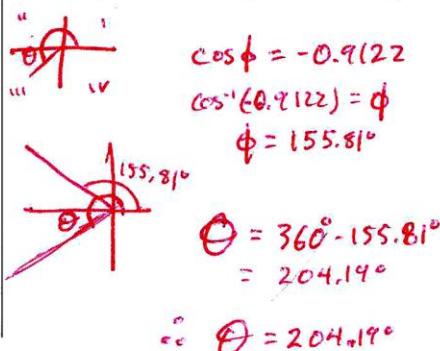


6. Determine the measure of angle θ in standard position, correct to one decimal place.

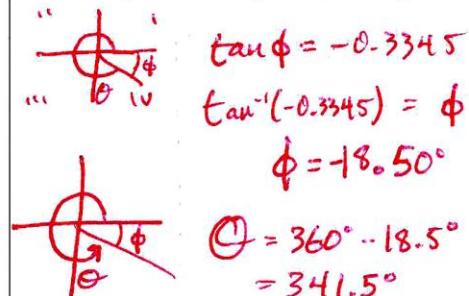
- a) $\sin\theta = 0.8660$, if θ is in the second quadrant. [K: 2 marks]



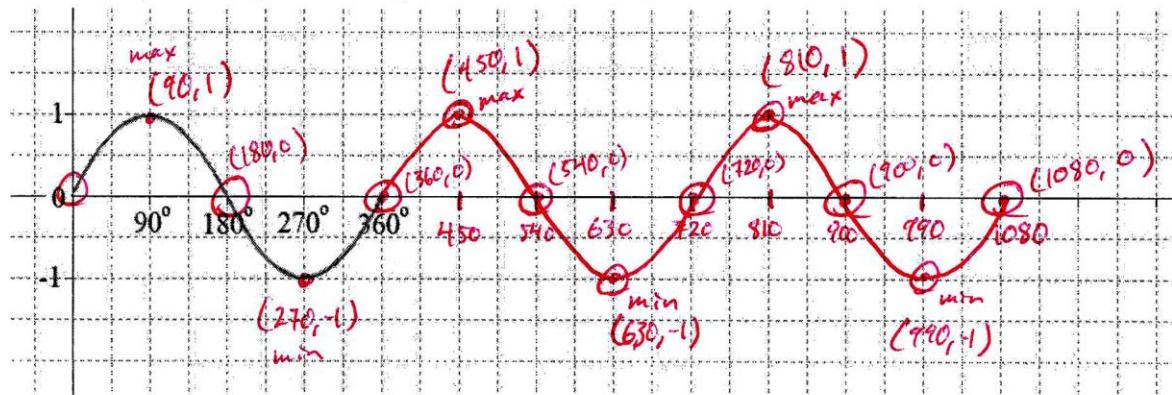
- b) $\cos\theta = -0.9122$, if θ is in the third quadrant. [K: 2 marks]



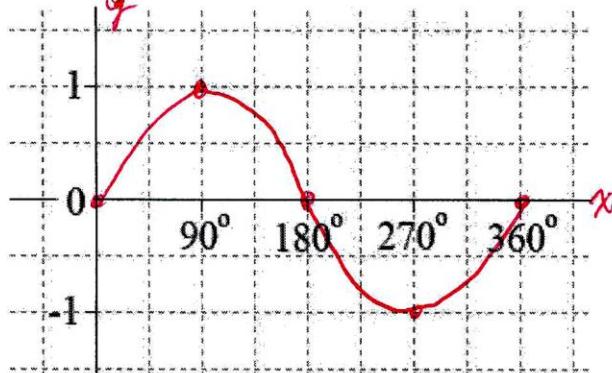
- c) $\tan\theta = -0.3345$, if θ is in the fourth quadrant. [K: 2 marks]



7. Extend the graph for two more periods. Label all intercepts and all maximum and minimum points. [C: 3 marks]



8.



- b) Locate all the points where $y = 1$ and give the values of x . [C: 1 mark]

$$x = 90^\circ$$

- c) Locate all the points where $y = -1$ and give the values of x [C: 1 mark]

$$x = 270^\circ$$

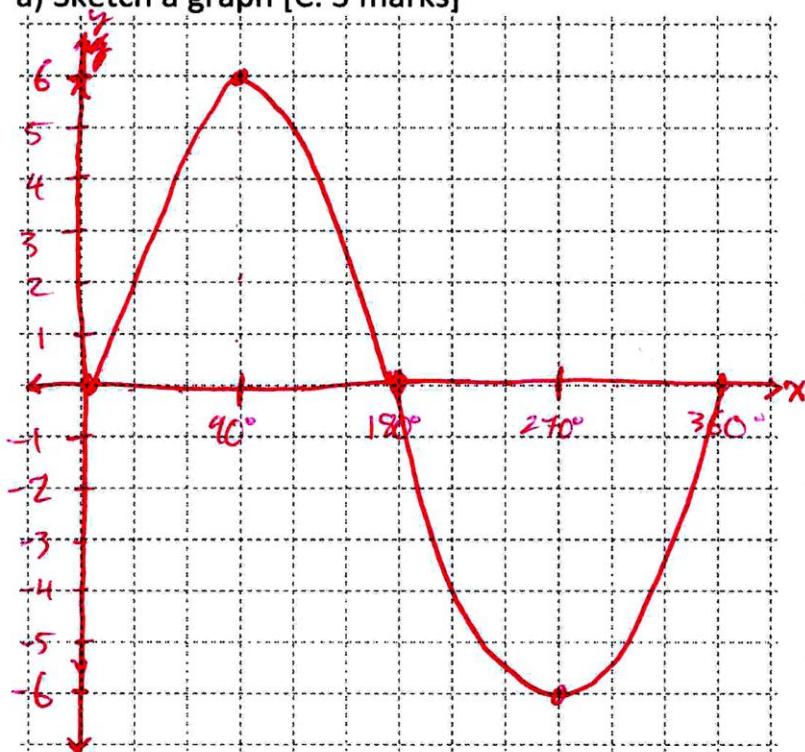
- a) Draw a sketch of $y = \sin(x)$ for one period. [C: 2 marks]

- d) Locate all the points where $y = 0$ and give the values of x [C: 2 marks]

$$x = 0^\circ, 180^\circ, 360^\circ$$

9. For the function $y = 6\sin(x)$ where $0^\circ \leq x \leq 360^\circ$:

a) Sketch a graph [C: 3 marks]



b) Determine the period. [K: 1 mark]

$$P = 360^\circ$$

c) Determine the amplitude [K: 1 mark]

$$a = 6$$

d) Determine the domain [C: 1 mark]

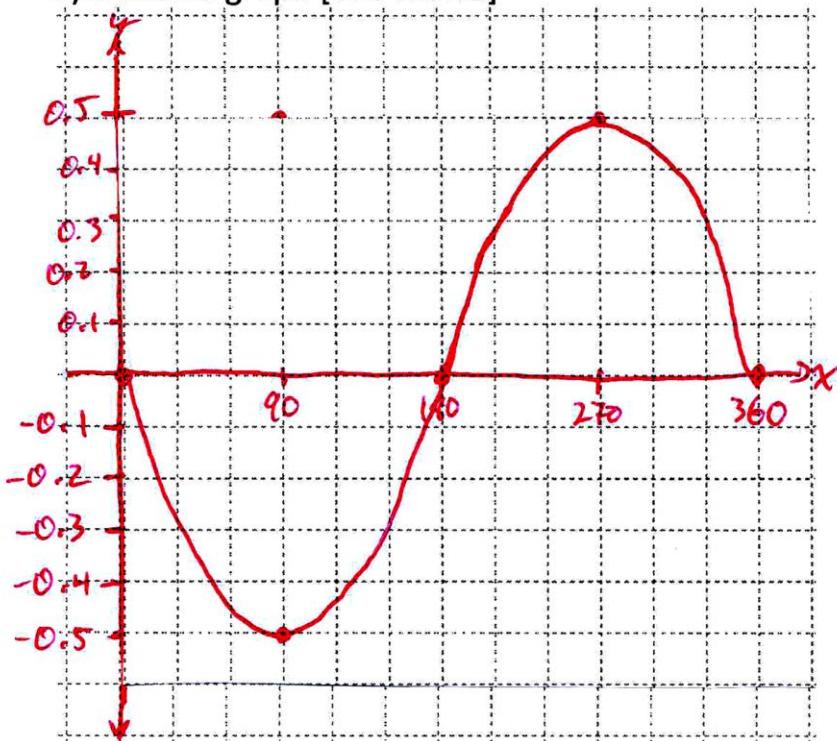
$$\{x \in \mathbb{R}, 0 \leq x \leq 360\}$$

e) Determine the range [C: 1 mark]

$$\{y \in \mathbb{R}, -6 \leq y \leq 6\}$$

10. For the function $y = -\frac{1}{2}\sin(x)$ where $0^\circ \leq x \leq 360^\circ$:

a) Sketch a graph [C: 3 marks]



b) Determine the period. [K: 1 mark]

$$\text{Period} = 360^\circ$$

c) Determine the amplitude [K: 1 mark]

$$a = 0.5$$

d) Determine the domain [C: 1 mark]

$$\{x \in \mathbb{R}, 0 \leq x \leq 360\}$$

e) Determine the range [C: 1 mark]

$$\{y \in \mathbb{R}, -0.5 \leq y \leq 0.5\}$$

$$y = a \sin(x-d) + c$$

11. Write an equation for the sine function [T: 3 marks]

$$a = \frac{\max - \min}{2} = \frac{4-2}{2} = \frac{2}{2} \\ = 1$$

Using point $(0, 3)$

$$3 = \sin(0) + c$$

$$3 = 0 + c$$

$$c = 3$$

Using point $(90^\circ, 4)$

$$4 = \sin(90^\circ - d) + 3$$

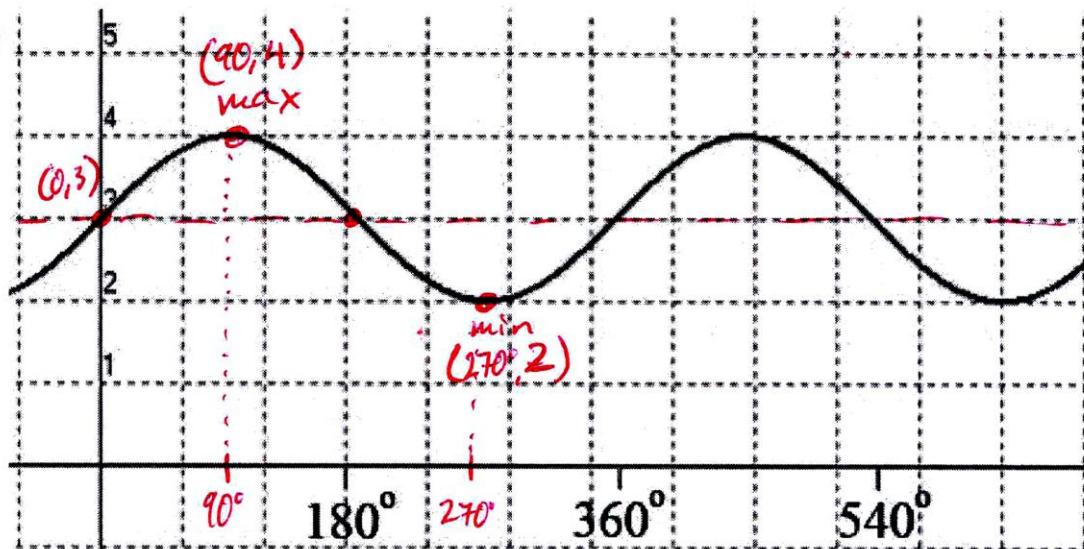
$$4-3 = \sin(90^\circ - d)$$

$$1 = \sin(90^\circ - d)$$

$$\sin^{-1}(1) = 90^\circ - d$$

$$90^\circ = 90^\circ - d$$

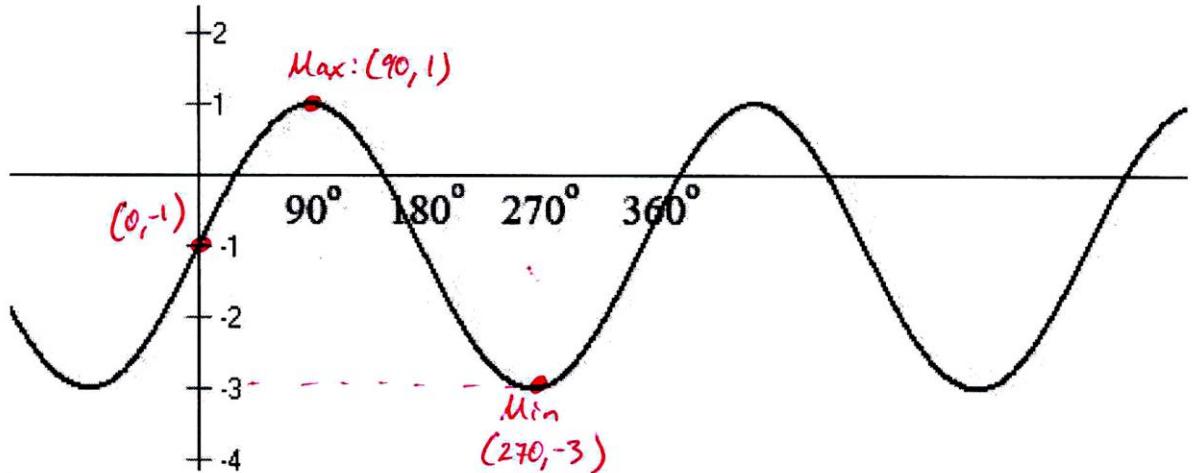
$$d = 0$$



$$\therefore y = \sin(x) + 3$$

12. Write an equation for the sine function [T: 3 marks]

$$a = \frac{\max - \min}{2} \\ = \frac{1 - (-3)}{2} \\ = \frac{4}{2} \\ = 2 \\ \text{so } a = 2$$



Using point $(0, -1)$

$$-1 = (2) \sin(0) + c$$

$$-1 = (2)(0) + c$$

$$-1 = 0 + c$$

$$\text{so } c = -1$$

Using point $(90^\circ, 1)$

$$1 = 2 \sin(90^\circ - d) + 1$$

$$1+1 = 2 \sin(90^\circ - d)$$

$$\frac{2}{2} = \sin(90^\circ - d)$$

$$1 = \sin(90^\circ - d)$$

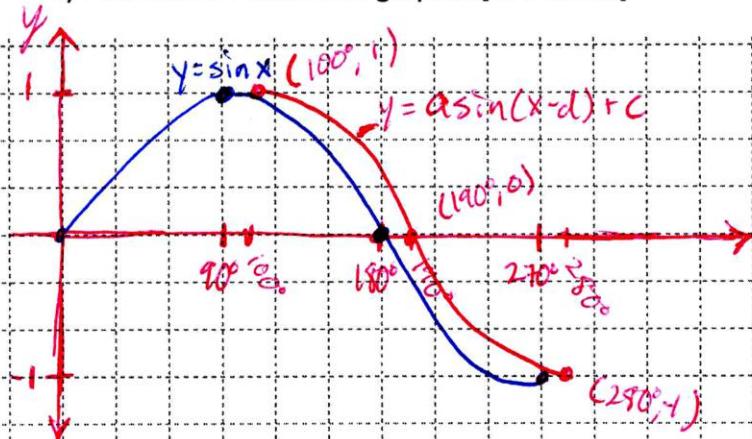
$$\sin^{-1}(1) = 90^\circ - d$$

$$90^\circ = 90^\circ - d \quad \text{so } d = 0$$

$$\therefore y = 2 \sin(x) - 1$$

13. The graph of a sine curve passes through the points $(100^\circ, 1)$, $(190^\circ, 0)$, $(280^\circ, -1)$.

a) Plot the sine curve and graph it [C: 3 marks]



b) Determine an equation that represents this function.
[T: 4 marks]

We see from the graph that the red line, $y = \sin(x-d)+c$, has an amplitude of 1, so $a=1$. The red line also has the exact same shape as the blue line, $y=\sin(x)$ but shifted to the right by 10 degrees, so $d=10$. The middle of the red line is at $y=0$, so $c=0$.

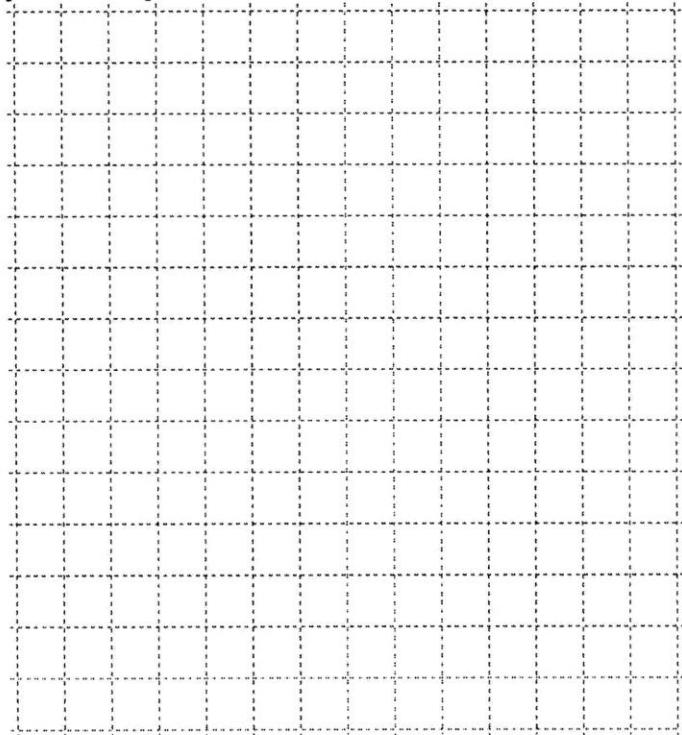
$$\text{Therefore, } y = \sin(x-10)$$

14. A satellite orbits Earth such that its displacement (distance north or south) from the equator (ignoring altitude) is given as

$$y = 7200 \sin(1.43t - 14.32)$$

Unit ↓ , where t is the time (in minutes), and y is the distance (in kilometres).

a) Sketch a graph of the function for 500 min.
[C: 3 marks]



b) What is the displacement from the equator after 1 h? [A: 2 marks]

$$1h = 60 \text{ min} \text{ so } t = 60$$

$$\begin{aligned} y &= 7200(\sin(1.43 \cdot 60 - 14.32)) \\ &= 7200 \sin(71.48) \\ &= 6827.13 \end{aligned}$$

∴ the displacement is 6827.13 m. north of the equator

c) What is the displacement from the equator after 11 h? [A: 2 marks]

$$11h = 660 \text{ min} \text{ so } t = 660$$

$$\begin{aligned} y &= 7200 \sin(1.43 \cdot 660 - 14.32) \\ &= 7200 \sin(929.48) \\ &= -3543.26 \end{aligned}$$

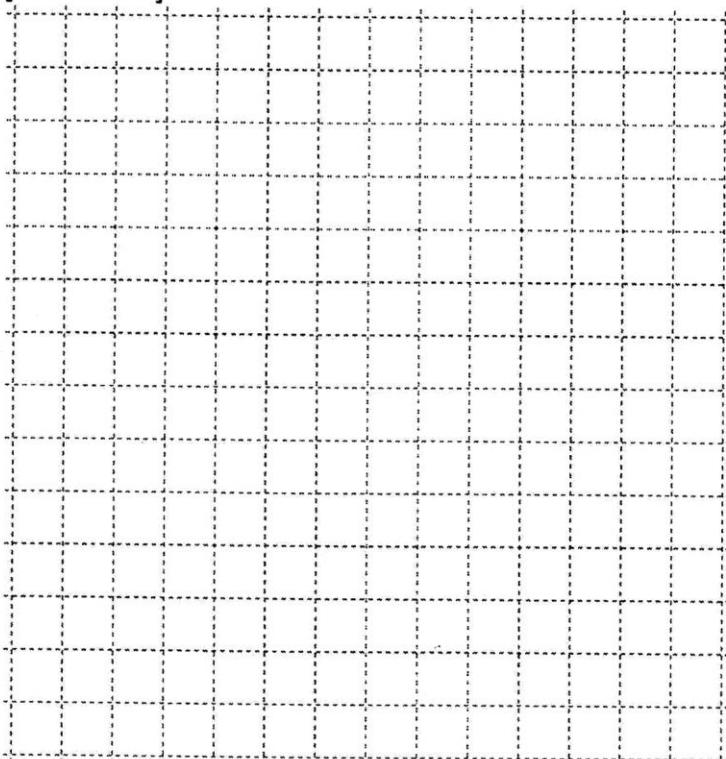
∴ The displacement is 3543.26 m. south of the equator

15. The electric current, i , in microamperes (μA), in a circuit is given by the equation

$$i = 4 \sin(360t + 11.5)$$

~~Omitted~~, where t is the time (in seconds).

- a) Sketch a graph of the function for 500 min.
[C: 3 marks]



- b) What is the amplitude of the current? [A: 2 marks]

$$\alpha = 4 \text{ so...}$$

The amplitude is $4 \mu\text{A}$

- c) What is the current at the start?
[A: 2 marks]

$$t = 0$$

$$\begin{aligned} i &= 4 \sin(360 \times 0 + 11.5) \\ &= 4 \sin(11.5) \\ &= 0.797 \end{aligned}$$

∴ The current is $0.797 \mu\text{A}$ at the start.

- d) What is the current after 1h?

$$[A: 2 \text{ marks}] \quad 1 \text{ hr} = 60 \text{ min} \quad 60 \text{ min} = 3600 \text{ s}$$

$$t = 3600 \text{ s}$$

$$\begin{aligned} i &= 4 \sin(360 \times 3600 + 11.5) \\ &= 4 \sin(1296000 + 11.5) \\ &= 0.797 \mu\text{A} \end{aligned}$$

16. Choose a question from #1-14 that you are confident in answering correctly. You will teach how to solve this question in front of the entire class. [C: 5 marks]

[1 mark for correct answer(s), 1 mark for clarity in voice, 1 mark for clarity in writing, 1 mark for extemporaneous quality, 1 mark for preparedness]