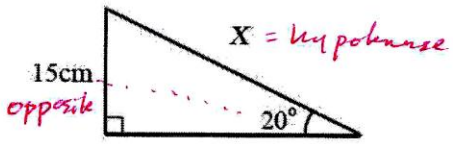
Solutions

MCF 3M - Mr. Choi mr-choi.weebly.com Due: July 8, 2013

Assignment 4

1. Find the length of *x*. Round to the nearest unit.

a) [K: 2 marks]

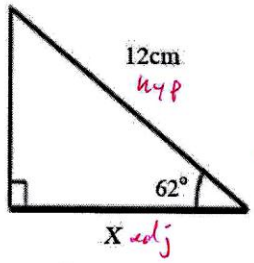


$\sin\theta = \frac{O}{H}$

$\sin 20 = \frac{15cm}{x}$

$X = \frac{15cm}{\sin 20} = 43.86cm$

b) [K: 2 marks]



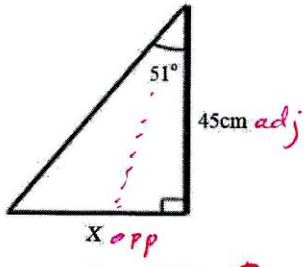
$\cos\theta = \frac{A}{H}$

$\cos(62) = \frac{x}{12cm}$

$X = 12cm \times \cos 62$

$X = 5.63cm$

c) [K: 2 marks]



$\tan\theta = \frac{O}{A}$

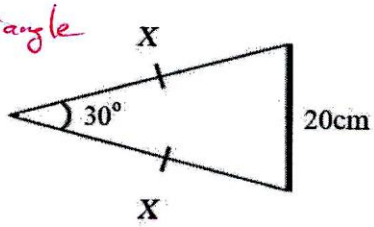
$\tan(51) = \frac{x}{45cm}$

$X = \tan(51) \times 45cm$

$X = 55.6cm$

2. A piece of metal is to be cut into an isosceles triangular shape. The unequal side is to be 30cm in length and is to be opposite a 15° angle. What are the lengths of the other two sides? [T: 3 marks]

Cut the triangle in half to create a right-triangle



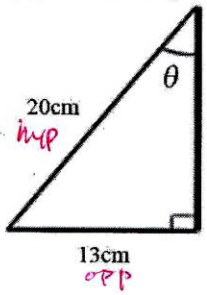
$\sin(15) = \frac{10cm}{X}$

$X = \frac{10cm}{\sin(15)} = 38.64cm$

∴ The lengths of the other 2 sides are both 38.64cm.

3. Find the measure of the marked angle, ϑ, to the nearest degree.

a) [K: 2 marks]

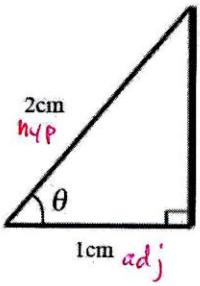


$\sin\theta = \frac{O}{H}$ so $\sin^{-1}\left(\frac{O}{H}\right) = \theta$

$\sin^{-1}\left(\frac{13}{20}\right) = \theta$

$\theta = 40.54°$

b) [K: 2 marks]



$\cos\theta = \frac{A}{H}$

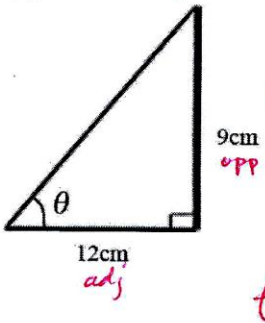
so

$\cos^{-1}\left(\frac{A}{H}\right) = \theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \theta$

$\theta = 60°$

c) [K: 2 marks]



$\tan\theta = \frac{O}{A}$

so

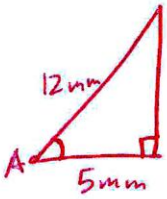
$\tan^{-1}\left(\frac{O}{A}\right) = \theta$

$\tan^{-1}\left(\frac{9}{12}\right) = \theta$

$\theta = 36.87°$

4. Side AC has a length of 10mm and side AB has a length of 12mm. Find ∠BAC to the nearest degree. [T: 3 marks]

Cut triangle ABC in half

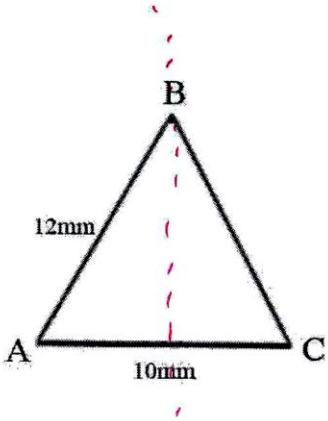


∠A shows that 5mm is our adjacent side and 12mm is hypotenuse

so $\cos^{-1}\left(\frac{5}{12}\right) = A$

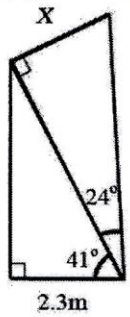
$A = 65.38°$

∴ ∠BAC is 65°.



5. Find the length of *x*

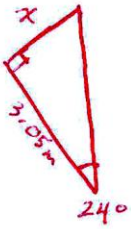
a) [T: 3 marks] Split the two triangles and work them separately



$\cos\theta = \frac{A}{H}$

$\cos(41°) = \frac{2.3m}{H}$

$H = \frac{2.3m}{\cos(41°)} = 3.05m$



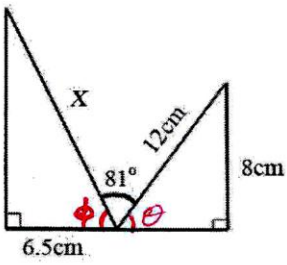
$\tan\theta = \frac{O}{A}$

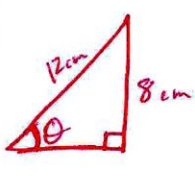
$\tan(24°) = \frac{x}{3.05m}$

$x = \tan(24°) \times 3.05m$

$x = 1.36m$

b) [T: 3 marks] Solve triangles separately. Solve the one on the right first





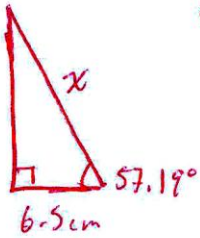
$\sin\theta = \frac{O}{H}$ so $\sin^{-1}\left(\frac{O}{H}\right) = \theta$

$\theta = \sin^{-1}\left(\frac{8}{12}\right)$

$\theta = 41.81°$

$\phi = 180 - 81 - 41.81$

$\phi = 57.19°$



$\cos\phi = \frac{A}{H}$

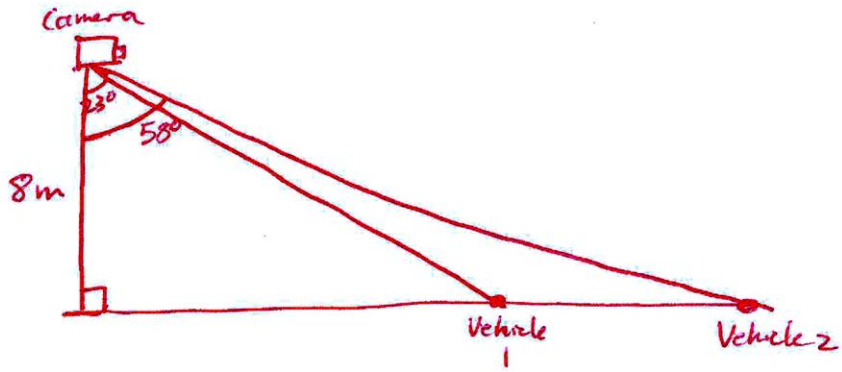
$\cos(57.19) = \frac{6.5cm}{x}$

$x = \frac{6.5cm}{\cos(57.19)} = 12cm$

$x = 12cm$

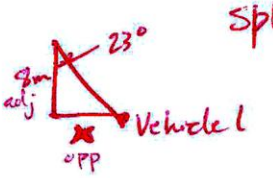
6. A security camera is mounted 8m above the ground on the outside front wall of a building. Police investigators used the camera to determine that the angles of depression to two vehicles parked directly in line with the camera were 23° and 58°.

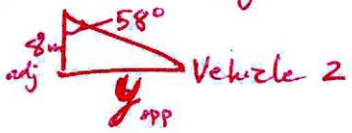
a) Draw a diagram to model the problem. List the given angles and measurements in your diagram. [C: 4 marks]



b) How far apart were the vehicles, to the nearest tenth of a metre? [A: 5 marks]

Split the above triangle into 2





$\tan\theta = \frac{O}{A}$

$\tan(23°) = \frac{x}{8m}$

$8m \times \tan(23°) = x$

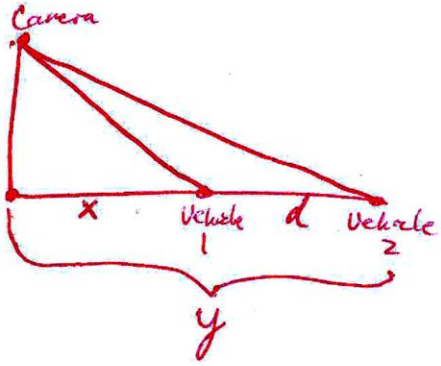
$x = 3.40m$

$\tan\theta = \frac{O}{A}$

$\tan(58) = \frac{y}{8m}$

$8m \times \tan(58) = y$

$y = 12.80\ m$



The distance between the cars is $y - x$

$d = y - x$

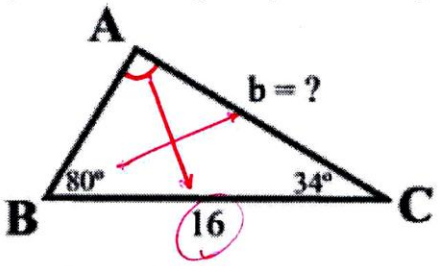
$= 12.80m - 3.40m$

$= 9.4m$

∴ The vehicles are 9.4m apart.

7. Solve for the following measurements:

a) Solve for length b [K: 2 marks]



Solve Angle A first

$A = 180 - 80 - 34$

$= 66°$

Use sine law

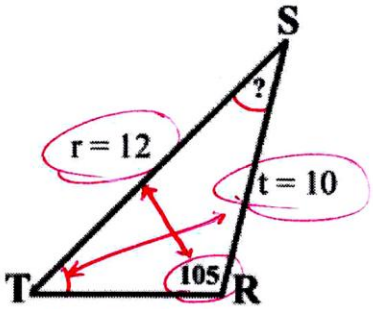
$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 80} = \frac{16}{\sin 66}$

$b = \frac{16}{\sin 66} \times \sin 80$

$b = 17.25$

b) Solve for angle S [K: 2 marks]



Solve Angle T first using Sine Law

$\frac{\sin T}{10} = \frac{\sin 105}{12}$

$\sin T = \frac{\sin 105}{12} \times 10$

$\sin T = 0.805$

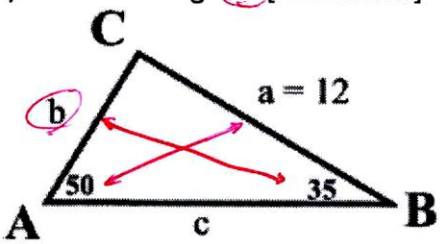
$T = \sin^{-1}(0.805)$

$T = 53.604$

so $S = 180° - 105° - 53.6°$

$S = 21.4°$

c) Solve for length b [K: 2 marks]



Use Sine Law

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{b}{\sin 35} = \frac{12}{\sin 50}$

$b = \frac{12}{\sin 50} \times \sin 35$

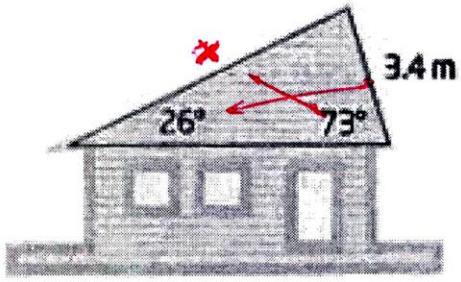
$b = 8.99$

8. The roof of a building makes angles of 26° and 73° with the horizontal. The shorter roof rafter is 3.4m long. Determine the length of the longer roof rafter, rounded to the nearest tenth of a metre. A diagram of the house is seen below. [A: 2 marks]

$\frac{x}{\sin(73)} = \frac{3.4m}{\sin(26)}$

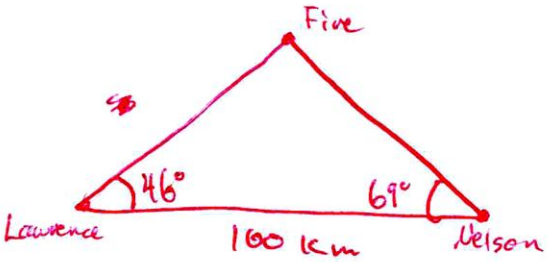
$x = \frac{3.4}{\sin(26)} \times \sin(73)$

$x = 7.42m$



9. Two ranger stations are 100km apart. A fire is located between the two ranger stations at an angle of 46° from Lawrence Station and 69° from Nelson Station.

a) Draw a diagram [C: 3 marks]

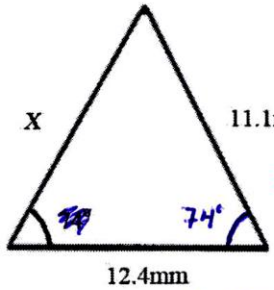


b) Which station is closer to the fire? [C: 1 marks]

Nelson station is closer because its length is opposite to the smaller angle and smaller lengths are always opposite to the smaller angle in a triangle.

10. Find the length x in part a, and angle ϑ in part b.

a) [K: 3 marks] * Question has been changed *



$x^2 = 11.1^2 + 12.4^2 - 2(11.1)(12.4)\cos(74°)$

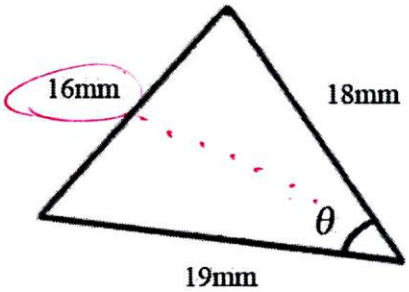
$x^2 = 123.21 + 153.76 - 75.877$

$x^2 = 201.09$

$x = \sqrt{201.09}$

$x = 14.18$

b) [K: 3 marks]



$16^2 = 18^2 + 19^2 - 2(18)(19)\cos\theta$

$256 = 324 + 361 - 684\cos\theta$

$256 = 685 - 684\cos\theta$

$256 - 685 = -684\cos\theta$

$\frac{-429}{-684} = \frac{-684\cos\theta}{-684}$

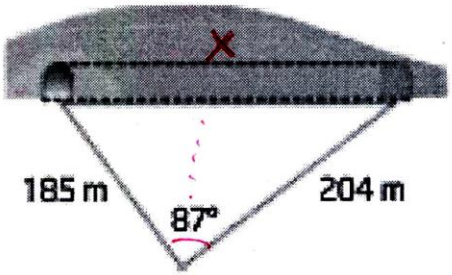
$0.627 = \cos\theta$

$\cos^{-1}(0.627) = \theta$

$\theta = 51.2°$

11. In order to plan a tunnel through a mountain, a surveyor makes the measurements shown. Use the surveyor's measurements to determine the length of the tunnel to the nearest metre [A: 3 marks]

Let's call x the length of the tunnel.



$x^2 = 204^2 + 185^2 - 2(204)(185)\cos(87°)$

$x^2 = 41\,616 + 34\,225 - 3950.32$

$x^2 = 71\,890.68$

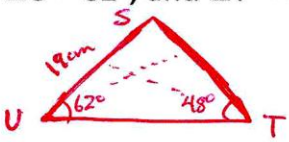
$x = \sqrt{71\,890.68}$

$x = 268.12\text{m}$

$\therefore$ The length of the tunnel is 268.12 m.

12. Draw then solve each triangle. Round your answers to the nearest tenth of a unit. [C: 3 marks] [T: 2 marks] each.

a) In ΔSTU, SU = 19cm, ∠U = 62°, and ∠T = 48°.



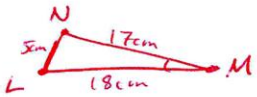
$\angle S = 180° - 62° - 48°$
$= 70°$

$\frac{ST}{\sin(62)} = \frac{19}{\sin(48)}$

$ST = \frac{19}{\sin(48)} \times \sin(62)$
$= 22.57\text{cm}$

$\frac{UT}{\sin(70)} = \frac{19}{\sin(48)}$
$UT = \frac{19}{\sin(48)} \times \sin(70)$
$= 24.03\text{cm}$

b) In ΔLMN, LM = 18cm, LN = 5cm, MN = 17cm.



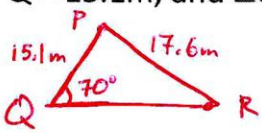
∠M: Use Cosine Law to find

$5^2 = 17^2 + 18^2 - 2(17)(18)\cos M$
$25 = 289 + 324 - 612\cos M$
$25 = 613 - 612\cos M$
$25 - 613 = -612\cos M$
$\frac{-588}{-612} = \frac{-612\cos M}{-612}$
$0.961 = \cos M$
$M = \cos^{-1}(0.961)$
$= 16.1°$

$\frac{\sin L}{17} = \frac{\sin(16.1)}{5}$
$\sin L = \frac{\sin(16.1)}{5} \times 17$
$\sin L = 0.94$
$L = \sin^{-1}(0.94)$
$= 70.54°$

$180° - 70.54 - 16.1° = 93.3°$
$N = 93.3°$

c) In ΔPQR, PR = 17.6m, PQ = 15.1m, and ∠Q = 70°.



$\frac{\sin R}{15.1} = \frac{\sin(70)}{17.6}$

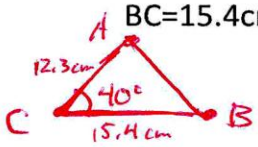
$\sin R = \frac{\sin(70)}{17.6} \times 15.1$

$\sin R = 0.806$
$R = \sin^{-1}(0.806)$
$R = 53.7°$

$P = 180° - 53.7 - 70$
$= 56.3°$

$\frac{QR}{\sin(56.3)} = \frac{17.6}{\sin(70)}$
$QR = \frac{17.6}{\sin(70)} \times \sin(56.3) = 15.6\text{m}$

d) In ΔABC, AC = 12.3cm, BC=15.4cm, and ∠C = 40°.



let c = AB

$c^2 = 12.3^2 + 15.4^2 - 2(12.3)(15.4)\cos(40)$
$= 151.29 + 237.16 - 290.21$
$= 98.24$

$c = \sqrt{98.24}$
$= 9.9\text{cm}$

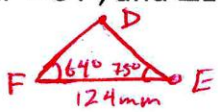
$\frac{\sin B}{12.3} = \frac{\sin 40}{9.90}$

$\sin B = \frac{\sin 40}{9.9} \times 12.3$

$\sin B = 0.7986$

$B = \sin^{-1}(0.7986) = 53°$ so $C = 180 - 53 - 40$
$= 87°$

e) In ΔDEF, EF = 124mm, ∠F = 64°, and ∠E = 75°.



$D = 180 - 64 - 75 = 41°$

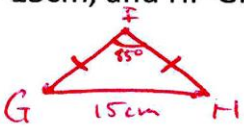
$\frac{DE}{\sin 64} = \frac{124}{\sin(41)}$

$DE = \frac{124}{\sin(41)} \times \sin 64$
$= 169.9\text{ mm}$

$\frac{FD}{\sin 75} = \frac{124}{\sin(41)}$

$FD = \frac{124}{\sin 41} \times \sin 75$
$= 182.6\text{ mm}$

f) In ΔGHI, ∠I = 85°, GH = 15cm, and HI=GI.



Because it's an isoceles triangle 2 sides are same length which means 2 angles are equal.

$\therefore \angle G = \angle H = \frac{180 - 85}{2}$

$\angle G = 47.5°$ and $\angle H = 47.5°$

$GI = HI$

$\frac{GI}{\sin(47.5)} = \frac{15}{\sin(85)}$

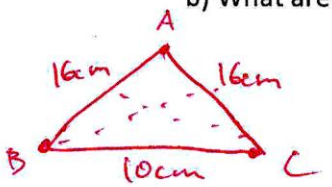
$GI = \frac{15}{\sin(85)} \times \sin(47.5)$
$= 11.1\text{ cm}$

so $HI = 11.1\text{cm}$

13. A steel plate is to have three holes drilled in it. Two of the holes are to be 10cm apart. The third hole is to be 16cm from each of the other two.

a) Draw a diagram to model this situation [C: 3 marks]

b) What are the angle measures formed by these three holes? [A: 3 marks]



$c^2 = a^2 + b^2 - 2ab\cos C$

$16^2 = 10^2 + 16^2 - 2(10)(16)\cos C$

$256 = 100 + 256 - 320\cos C$

$256 = 356 - 320\cos C$

$256 - 356 = -320\cos C$

$\frac{-100}{-320} = \frac{-320\cos C}{-320}$

$\cos C = 0.3125$

$C = \cos^{-1}(0.3125)$

$C = 71.8°$

Isoceles triangle means 2 sides are equal. If 2 sides are equal, their angles that lie opposite are equal.

So $\angle C = \angle B$

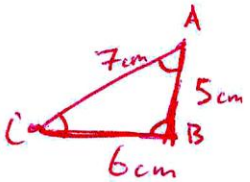
Thus $\angle B = 71.8°$

So $\angle A = 180 - 71.8 - 71.8$
$= 36.4°$

14. A triangular piece of wood is to be cut with side lengths of 5cm, 6cm, and 7cm.

a) Draw a diagram that the carpenter would draw. [C: 3 marks]

b) What angle measurements will the carpenter need to cut? Round answers to the nearest tenth of a degree. [A: 3 marks]



$c^2 = a^2 + b^2 - 2ab\cos C$

$5^2 = 6^2 + 7^2 - 2(6)(7)\cos C$

$25 = 36 + 49 - 84\cos C$

$25 = 85 - 84\cos C$

$25 - 85 = -84\cos C$

$-60 = -84\cos C$

$\cos C = \frac{-60}{-84} = 0.714$

$C = \cos^{-1}(0.714)$ ~~= 44.4°~~

$C = 44.4°$

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{6} = \frac{\sin(44.4)}{5}$

$\sin A = \frac{\sin(44.4)}{5} \times 6$

$\sin A = 0.8396$

$A = \sin^{-1}(0.8396)$

$A = 57.1°$

$B = 180° - 44.4° - 57.1°$
$= 78.5°$

15. Choose a question from #1-14 that you are confident in answering correctly. You will teach how to solve this question in front of the entire class. [C: 5 marks]

[1 mark for correct answer(s), 1 mark for clarity in voice, 1 mark for clarity in writing, 1 mark for extemporaneous quality, 1 mark for preparedness]