

Solutions

Assignment 3

1. Determine how many real roots each equation has.

a) $f(x) = 2x^2 + 11x + 5$ [K: 2 marks]

$$\Delta = 2x^2 + 11x + 5$$

$$D = (11)^2 - 4(2)(5)$$

$$= 121 - 40$$

$$= 81$$

$\because D > 0$ there are 2
real roots.

c) $x^2 - 4x = -4$ [K: 2 marks]

Rearrange: $x^2 - 4x + 4 = 0$

Then use

$$D = b^2 - 4ac$$

$$D = (-4)^2 - 4(1)(4)$$

$$= 16 - 16$$

$$= 0$$

$\because D = 0$ so there is
one real root

Use Discriminant
Formula

$$D = b^2 - 4ac$$

$$y = x^2 + x + 25$$

$$\Delta = x^2 + x + 25$$

$$D = (1)^2 - 4(1)(25)$$

$$= 1 - 100$$

$$= -99$$

$D < 0$ so there are
no real roots

d) $y = x^2 + x - 2$ [K: 2 marks]

$$\Delta = x^2 + x - 2$$

$$D = (1)^2 - 4(1)(-2)$$

$$= 1 + 8$$

$$= 9$$

$\because D > 0$ so there
are two real roots

2. Find the x-intercepts of the quadratic function. Round answers to the nearest hundredth. [K: 4 marks]

$$y = 2x^2 - x - 4$$

To find x-intercepts, ~~set~~
 $y = 0$.

$$\Delta = \underbrace{2x^2 - x - 4}_{\rightarrow (2)(-4)} = -8$$

products	sum
(1)(-8)	-7
(-1)(8)	7
(2)(4)	-2
(-2)(4)	2

} None work so you must
use Quadratic Formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1 + 32}}{4}$$

$$= \frac{1 \pm \sqrt{33}}{4}$$

$$x_1 = \frac{1 + \sqrt{33}}{4} = 1.69$$

$$x_2 = \frac{1 - \sqrt{33}}{4} = -1.19$$

∴ the x-intercepts
are $x = -1.19$
and $x = 1.69$

3. Rewrite each quadratic function into the vertex form then state the coordinate of the vertex and determine whether it's a maximum or minimum.

a) $f(x) = -2x^2 + 16x - 3$ [K: 5 marks]

Factor out
-2 from first
2 terms

$$\begin{aligned}
 &= -2(x^2 - 8x) - 3 \\
 &= -2(x^2 - 8x + 16 - 16) - 3 \\
 &= -2((x-4)^2 - 16) - 3 \\
 &= -2(x-4)^2 + \cancel{32} - 3 \\
 &= -2(x-4)^2 + 29
 \end{aligned}$$

$b = -8$ so $(\frac{b}{2})^2 = (-\frac{8}{2})^2 = (-4)^2 = 16$

a is a negative number because
 $a = -2$, so parabola opens down
making vertex a maximum

Vertex at $(4, 29)$ and it is a maximum.

b) $f(x) = \frac{1}{2}x^2 - 4x + 1$ [K: 5 marks]

Factor out
 $\frac{1}{2}$ from first
2 terms

$$\begin{aligned}
 &= \frac{1}{2}(x^2 - 8x) + 1 \\
 &= \frac{1}{2}(x^2 - 8x + 16 - 16) + 1 \\
 &= \frac{1}{2}((x-4)^2 - 16) + 1 \\
 &= \frac{1}{2}(x-4)^2 - 8 + 1 \\
 &= \frac{1}{2}(x-4)^2 - 7
 \end{aligned}$$

$b = -8$ so $(\frac{b}{2})^2 = (-\frac{8}{2})^2 = (-4)^2 = 16$

$a = \frac{1}{2}$ is a positive number
so parabola opens up
making the vertex a minimum.

Vertex is at $(4, -7)$ and it is a minimum

4. A missile is launched out of a silo buried in the ground. The height of the missile can be approximated by the equation

$$h(t) = -5t^2 + 35t - 30$$

Where $h(t)$ is the height, in metres, and t is the time, in seconds.

a) In which form is this quadratic function expressed? [A: 1 mark]

Because it's in form $y = ax^2 + bx + c$

The function is expressed in Vertex Form

b) Write the function in Factored Form [A: 3 marks]

$$\begin{aligned} h(t) &= -5t^2 + 35t - 30 \\ &= -5(t^2 - 7t + 6) \\ &= -5(t^2 - t - 6t + 6) \\ &= -5(t(t-1) - 6(t-1)) \\ &= -5(t-1)(t-6) \end{aligned}$$

$$\therefore h(t) = -5(t-1)(t-6)$$

c) Write the function in vertex form [A: 3 marks]

$$\begin{aligned} h(t) &= -5t^2 + 35t - 30 \\ &= -5(t^2 - 7t) - 30 \\ &= -5(t^2 - 7t + 12.25 - 12.25) - 30 \\ &= -5((t-3.5)^2 - 12.25) - 30 \\ &= -5(t-3.5)^2 + 61.25 - 30 \\ &= -5(t-3.5)^2 + 31.25 \end{aligned}$$

$$\therefore h(t) = -5(t-3.5)^2 + 31.25$$

Rewrite the equations obtained from your answers to the question below into the right boxes [A: 3 marks]

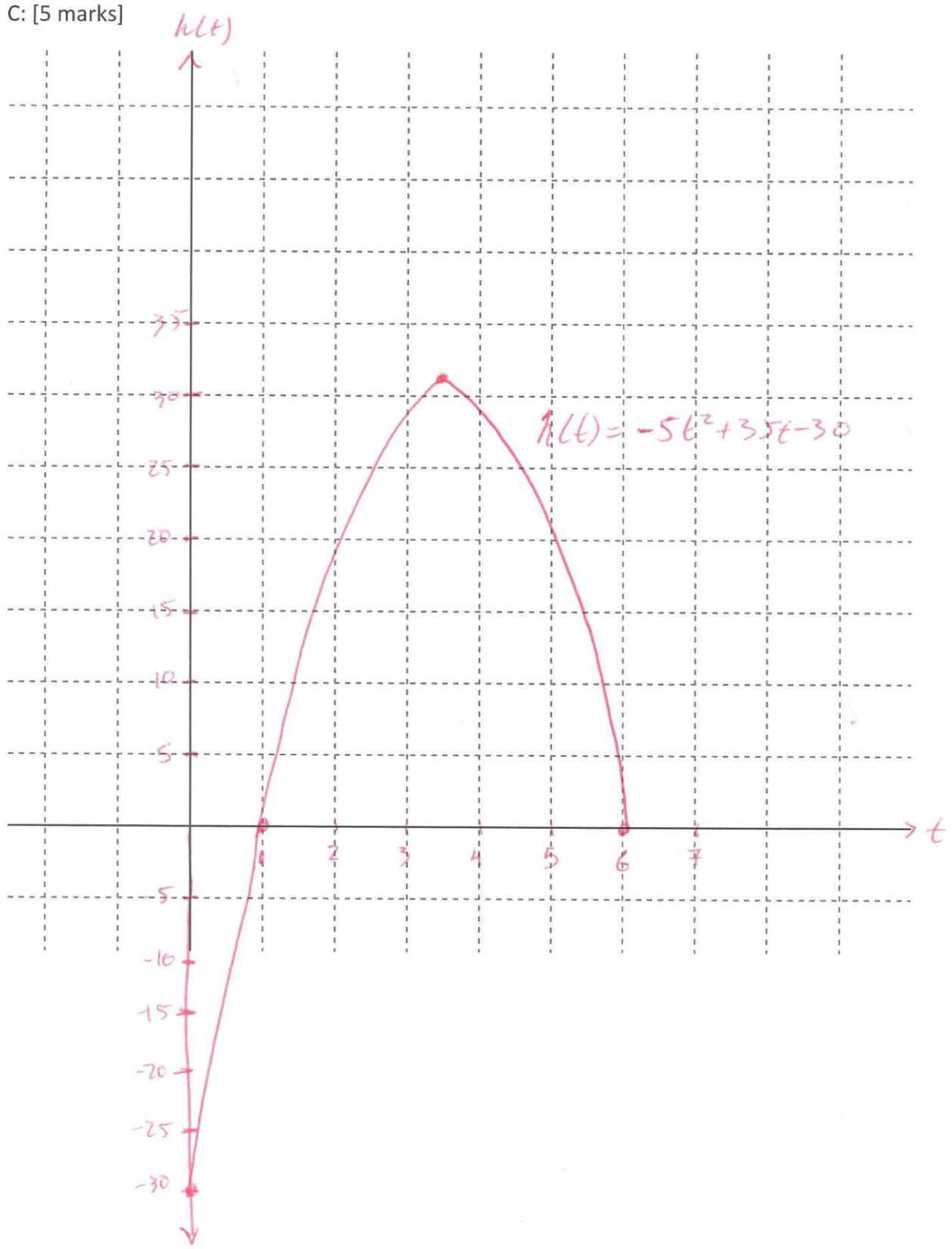
The function in Standard Form	The function in Factored Form	The function in Vertex form
$h(t) = -5t^2 + 35t - 30$	$h(t) = -5(t-1)(t-6)$	$h(t) = -5(t-3.5)^2 + 31.25$

Using the equations in the boxes above, fill out the boxes below:

c) The h -intercept [A: 1 mark] h(t) = -30	d) The zeros (aka: t -intercepts) of this function. [A: 2 marks] $t = 1$ and $t = 6$	e) The coordinates of the vertex. [A: 2 marks] (3.5) (3.5, 31.25)
f) How deep in the ground is the missile silo buried? [A: 1 mark] 30m below ground.	g) How long will it take for the missile to reach ground level? [A: 1 mark] $t = 1$ is when the missile reaches $h(t) = 0$ for the first time so... 1 second	h) What is the maximum height of the missile? [A: 1 mark] 31.25m
i) When does the missile hit the ground? [A: 1 mark] 6 seconds	j) How long does the missile take to reach its maximum height? [A: 1 mark] 3.5 seconds	k) How long is the missile in the air? [A: 1 mark] 6 seconds - 1 second = 5 seconds.

I) Graph the function of the missile using the answers you obtained from parts a) - k).
(Hint: You would plot the points that you used to help you answer parts c) to e))

C: [5 marks]



5. If one of the roots of $2x^2 + bx - 16 = 0$ is -8, find the values of b and the other root.

[T: 5 marks]

Since -8 is a root, we know that plugging $x = -8$ into the equation gets us 0, which can help us get the value of b.

$$2(-8)^2 + b(-8) - 16 = 0$$

$$2(64) - 8b - 16 = 0$$

$$128 - 8b - 16 = 0$$

$$-8b = -128 + 16$$

$$\frac{-8b}{-8} = \frac{-112}{-8}$$

$$b = +14$$

So with $b = +14$, we plug it into we see that

$$y = 2x^2 + 14x - 16$$

We can find the other root by factoring (or using quadratic equation)

$$y = 2(x^2 + 7x - 8)$$

$$= 2(x+8)(x-1)$$

$$x = -8 \quad x = 1$$

so $x = 1$ is the other root

6. Find a quadratic equation with each pair of roots

a) $x = \frac{14 \pm \sqrt{168}}{2}$ [T: 5 marks]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2a = 2 \text{ so } a = 1 \quad \text{use in}$$

$$-b = 14 \text{ so } b = -14$$

$$b^2 - 4ac = 168$$

$$(-14)^2 - 4(1)c = 168$$

$$196 - 4c = 168$$

$$-4c = 168 - 196$$

$$-4c = -28$$

$$c = \frac{-28}{-4}$$

$$c = 7$$

so...

$$\text{since } y = ax^2 + bx + c$$

$$\therefore y = x^2 - 14x + 7$$

b) $x = \frac{-11 \pm \sqrt{145}}{4}$ [T: 5 marks]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$2a = 4 \text{ so } a = 2 \quad \text{use in}$$

$$-b = -11 \text{ so } b = 11$$

$$b^2 - 4ac = 145$$

$$(11)^2 - 4(2)c = 145$$

$$121 - 8c = 145$$

$$-8c = 145 - 121$$

$$-8c = 24$$

$$c = \frac{24}{-8}$$

$$c = -3$$

so since $y = ax^2 + bx + c$

$$\therefore y = 2x^2 + 11x - 3$$