

Assignment 2

1. Expand and simplify

a) $3m(\underline{2m+9})$ [1 mark]

$$= 6m^2 + 27m$$

b) $4(x+2)^2 - (x+2)(x-2)$ [4 marks]

$$= 4(\underline{x+2})(\underline{x+2}) - (\underline{x+2})(\underline{x-2})$$

$$= 4(x^2 + 2x + 2x + 4) - (x^2 - 2x + 2x - 4)$$

$$= 4(\underline{x^2 + 4x + 4}) - (\underline{x^2 - 4})$$

$$= 4x^2 + \cancel{16x} + \cancel{16} - x^2 + 4$$

$$= 4x^2 - x^2 + \cancel{16x} + \cancel{16} + 4$$

$$= 3x^2 + \cancel{16x} + 20$$

2. Factor [2 marks]

a) $x^2 + 2x + 1$

$$= x^2 + x + x + 1$$

$$= x(x+1) + 1(x+1)$$

$$= (x+1)(x+1)$$

$$= (x+1)^2$$

$i \times 1 = 1$

Product

$(1)(1) = 1$

Sum

$1+1=2$

$(-1)(-1) = 1$

$(-1)+(-1) = -2$

b) $x^2 - x - 2$ [2 marks]

$i \times (-2) = -2$

$$= x^2 + x - 2x - 2$$

$$= x(x+1) - 2(x+1)$$

$$= (x+1)(x-2)$$

Product

$(1)(-2) = -2$

Sum

$1+(-2) = -1$

$(-1)(2) = -2$

$(-1)+2 = +1$

c) $4m^2 - 28m + 40$ [3 marks]

$$= 4(m^2 - 7m + 10)$$

$$= 4(m^2 - 2m - 5m + 10)$$

$$= 4(m(m-2) - 5(m-2))$$

$$= 4(m-2)(m-5)$$

$1 \times 10 = 10$

Product

$(1)(10) = 10$

Sum

$1+10=11$

$(-1)(-10) = 10$

Sum

$(-1)+(-10) = -11$

$(2)(5) = 10$

Product

$(2)(5) = 10$

Sum

$2+5=7$

$(-2)(-5) = 10$

Product

$(-2)(-5) = 10$

Sum

$(-2)+(-5) = -7$

d) $2a^2 + 12a + 16$ [3 marks]

$i \times 8 = 8$

$$= 2(\underline{a^2 + 6a + 8})$$

$$= 2(a^2 + 2a + 4a + 8)$$

$$= 2(a(a+2) + 4(a+2))$$

$$= 2(a+2)(a+4)$$

Product

$(1)(8) = 8$

Sum

$1+8=9$

$(-1)(-8) = 8$

Sum

$(-1)+(-8) = -9$

$(2)(4) = 8$

Product

$(2)(4) = 8$

Sum

$2+4=6$

$(-2)(-4) = 8$

Product

$(-2)(-4) = 8$

Sum

$(-2)+(-4) = -6$

3. Factor the expression further.

[1 mark]

$(x^2 + 4)(\underline{x^2 - 4})$

$\begin{matrix} p^2 \\ q^2 \\ p=1 \\ q=2 \end{matrix}$

Remember that

$$(px^2 - qx^2) = (px+q)(px-q)$$

$$= (x^2 + 4)(x+2)(x-2)$$

4. Consider the quadratic function

$$f(x) = -2(x - 1)^2 - 3$$

a) In which form is this function? Does this parabola open upward or downward?

Vertex form. The parabola opens downward because the a -value is negative.

b) What are the coordinates of the vertex? Is the vertex a maximum or minimum?

$h=1$ which is a shift right 1 unit
 $c=-3$ and shift down 3 units

so the vertex is at $(1, -3)$.
Since the parabola opens downward
 the vertex is a maximum.

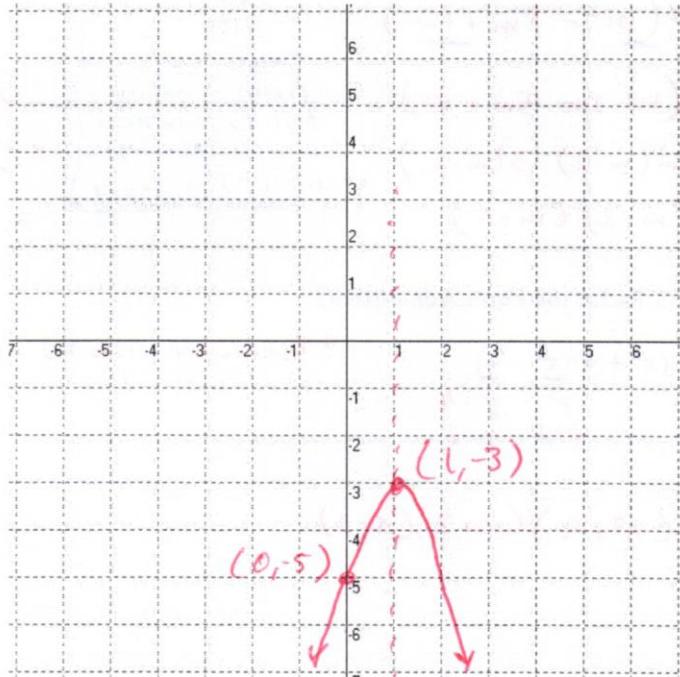
c) What is the axis of symmetry?

$$x = 1$$

d) What is the y-intercept?

$$\begin{aligned} f(0) &= -2(0-1)^2 - 3 \\ &= -2(-1)^2 - 3 \\ &= -2 - 3 \\ &= -5 \quad y = -5 \text{ is the } y\text{-int} \end{aligned}$$

e) Graph the function



5) Digital music player cases sell for \$20 each. Research by the seller has shown that for each \$10 increase in price, 100 fewer cases will be sold. The revenue generated from case sales can be represented by the function

$$R(x) = (2+x)(6-x)$$

where $R(x)$ is the revenue, in thousands of dollars, and x is the number of \$10 price increases.

a) What are the x -intercepts of this function? $R(x) = (2+x)(6-x)$

$$2+x=0 \quad 6-x=0$$

$$x=-2 \quad x=6$$

So the x -intercepts are $x=-2, x=6$

b) use the x -intercepts to find the coordinates of the vertex. *Remember, the vertex is in the middle*

$$X_{\text{mid}} = \frac{6+(-2)}{2} = \frac{6-2}{2} = \frac{4}{2} = 2$$

$$X_{\text{mid}} = 2$$

So the vertex is
at $(2, 16)$

c) Is the vertex a maximum or a minimum?

$$\begin{aligned} R(x) &= (2+x)(6-x) \\ &= 12 - 2x + 6x - x^2 \\ &= \textcircled{-}x^2 + 4x + 12 \end{aligned}$$

α is negative so parabola opens down
so the vertex is a maximum.

d) How many \$10 price increases should there be in order to maximize revenue?

Vertex is maximum so since x represents the number of \$10 increases, there should be 2 ~~no~~ price increases.

e) What is the selling price that gives the maximum revenue?

Price is originally \$20, but with 2 price increases of \$10
~~the~~ the selling price that obtains maximum revenue
is \$40.

6) A javelin is thrown from a raised platform. Its height as a function of time is given by the equation:

$$h(t) = -5t^2 + 20t + 25$$

, where $h(t)$ is the height of the javelin, in metres, and t is the time, in seconds.

a) Write the equation in factored form.

$$\begin{aligned} h(t) &= -5t^2 + 20t + 25 \\ &= -5(1t^2 - 4t - 5) \\ &= -5(t^2 + t - 5t - 5) \\ &= -5(t(t+1) - 5(t+1)) \\ &= -5(t+1)(t-5) \end{aligned}$$

$$\begin{array}{rcl} 1 \times (-5) &=& -5 \\ \text{products} & & \text{sum} \\ \hline (1)(-5) & & 1 + (-5) = -4 \\ (-1)(5) & & (-1) + 5 = 4 \end{array}$$

b) What is the hang time of the javelin?

Hang time of javelin means from $t=0$ to the t -value of when the javelin hits the ground.

Javelin hitting the ground is when $h(t)=0$.

$$h(t) = 0 = -5(t+1)(t-5)$$

$$\begin{array}{l} t+1=0 \\ \cancel{t=-1} \end{array}$$

$$\begin{array}{l} t-5=0 \\ t=5 \end{array}$$

Factored form helps us get the "zeros" or "roots"

We choose this one because the other t is negative which doesn't make sense (you can't have negative time).

the javelin is launched at $t=0$

and Javelin hits ground after 5s ...

~~so hang time~~

∴ The hang time of javelin is 5 seconds.