KEY CONCEPTS

- A polynomial expression has the form
  $a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_3x^3 + a_2x^2 + a_1x + a_0$
  where
  - n is a whole number
  - x is a variable
  - the coefficients $a_0, a_1, \ldots, a_n$ are real numbers
  - the degree of the function is n, the exponent of the greatest power of x
  - a_n, the coefficient of the greatest power of x, is the leading coefficient
  - a_0, the term without a variable, is the constant term
- A polynomial function has the form
  $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$
- A power function is a polynomial of the form $y = ax^n$, where n is a whole number.
- Power functions have similar characteristics depending on whether their degree is even or odd.
- Even-degree power functions have line symmetry in the y-axis, $x = 0$.
- Odd-degree power functions have point symmetry about the origin, (0, 0).

Communicate Your Understanding

C1 Explain why the function $y = 3$ is a polynomial function.

C2 How can you use a graph to tell whether the leading coefficient of a power function is positive or negative?

C3 How can you use a graph to tell whether the degree of a power function is even or odd?

C4 State a possible equation for a power function whose graph extends

a) from quadrant 3 to quadrant 1

b) from quadrant 2 to quadrant 1

c) from quadrant 2 to quadrant 4

d) from quadrant 3 to quadrant 4

Complete Questions #1 - 7

A Practise

For help with questions 1 and 2, refer to Example 1.

1. Identify whether each is a polynomial function. Justify your answer.

a) $p(x) = \cos x$ **b)** $h(x) = -7x$

c) $f(x) = 2x^4$ **d)** $y = 3x^5 - 2x^3 + x^2 - 1$

e) $k(x) = 8^x$ **f)** $y = x^{-3}$

2. State the degree and the leading coefficient of each polynomial.

a) $y = 5x^4 - 3x^3 + 4$ **b)** $y = -x + 2$

c) $y = 8x^2$ **d)** $y = -\frac{x^3}{4} + 4x - 3$

e) $y = -5$ **f)** $y = x^2 - 3x$

For help with question 3, refer to Example 2.

3. Consider each graph.

i) Does it represent a power function of even degree? odd degree? Explain.

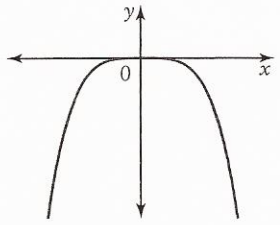
ii) State the sign of the leading coefficient. Justify your answer.

iii) State the domain and range.

iv) Identify any symmetry.

v) Describe the end behaviour.

a)



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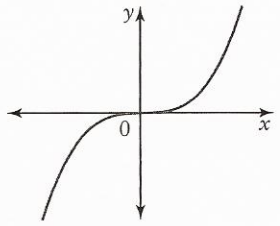

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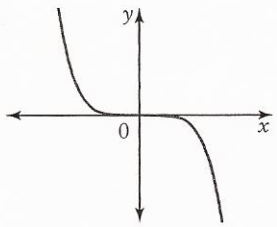

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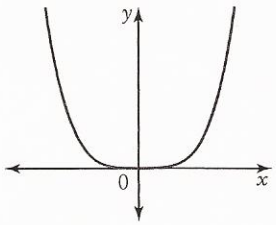

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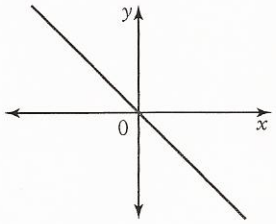


For help with question 4, refer to Example 3.

4. Copy and complete the table for the following functions.

$y = -x^3$ $\quad y = \frac{3}{7}x^2$ $\quad y = 5x$

$y = 4x^5$ $\quad y = -x^6$ $\quad y = -0.1x^{11}$

$y = 2x^4$ $\quad y = -9x^{10}$

End Behaviour	Function	Reasons
Extends from quadrant 3 to quadrant 1		
Extends from quadrant 2 to quadrant 4		
Extends from quadrant 2 to quadrant 1		
Extends from quadrant 3 to quadrant 4		

B Connect and Apply

For help with questions 5 and 6, refer to Example 4.

5. As a tropical storm intensifies and reaches hurricane status, it takes on a circular shape that expands outward from the eye of the storm. The area, A, in square kilometres, spanned by a storm with radius, r, in kilometres, can be modelled by the function $A(r) = \pi r^2$.

a) Graph $A(r)$ for $r \in [0, 10]$.

b) State the domain and range.

c) Describe the similarities and differences between the graph of $A(r)$ and the graph of $y = x^2$.

6. The circumference, C, in kilometres, of the tropical storm in question 5 can be modelled by the function $C(r) = 2\pi r$.

a) Graph $C(r)$ for $r \in [0, 10]$.

b) State the domain and range.

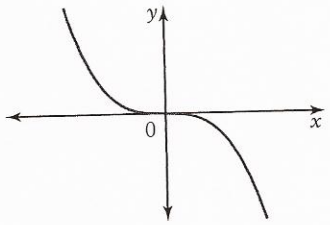
c) Describe the similarities and differences between the graph of $C(r)$ and the graph of $y = x$.

7. Determine whether each graph represents a power function, an exponential function, a periodic function, or none of these. Justify your choice.

CONNECTIONS

You worked with periodic functions when you studied trigonometric functions in grade 11. Periodic functions repeat at regular intervals.

a)



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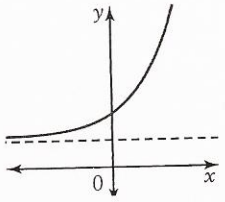

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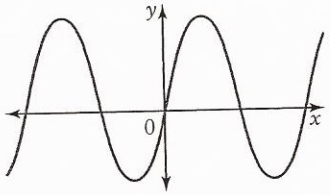

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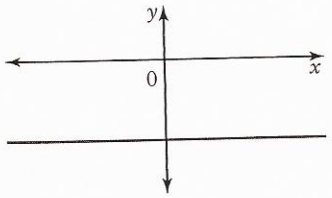

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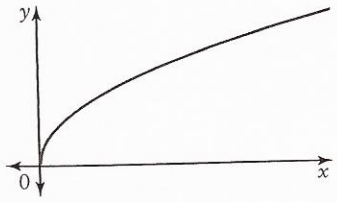

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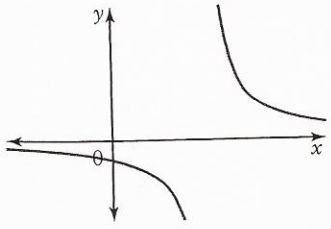

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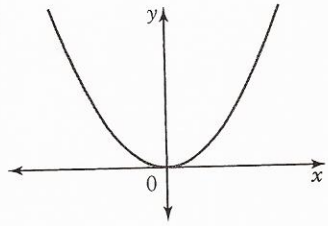
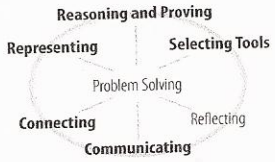


8. Use Technology



a) Graph $f(x) = x^3 + x^2$, $g(x) = x^3 - x$, and $h(x) = x^3$ on the same set of axes.

b) Compare and describe the key features of the graphs of these functions.

9. Use Technology

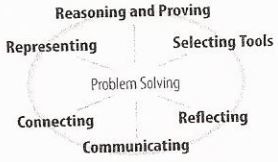
a) Graph $f(x) = x^4 + x$, $g(x) = x^4 - x^2$, and $h(x) = x^4$ on the same set of axes.

b) Compare and describe the key features of the graphs of these functions.

10. Describe the similarities and differences between the line $y = x$ and power functions with odd degree greater than one. Use graphs to support your answer.

11. Describe the similarities and differences between the parabola $y = x^2$ and power functions with even degree greater than two. Use graphs to support your answer.

12. a) Graph the functions $y = x^3$, $y = x^3 - 2$, and $y = x^3 + 2$ on the same set of axes. Compare the graphs. Describe how the graphs are related.



b) Repeat part a) for the functions $y = x^4$, $y = x^4 - 2$, and $y = x^4 + 2$.

c) Make a conjecture about the relationship between the graphs of $y = x^n$ and $y = x^n + c$, where $c \in \mathbb{R}$ and n is a whole number.

d) Test the accuracy of your conjecture for different values of n and c.