Assignment 7

1. a) Calculate the simple interest earned on an investment of $5000 earning 2.85% per year for 30 months. [K: 2 marks]

$P = \$5000$ $r = 0.0285$ per year

$t = \frac{30 \text{ months}}{12 \frac{\text{months}}{\text{year}}} = 2.5$ years

$I = Prt$

$= (5000)(0.0285)(2.5)$

$= 356.25$

∴ Simple interest earned is $356.25

b) Determine the final amount of the investment
[K: 1 mark]

$A = P + I$

$= 5000 + 356.25$

$= 5356.25$

∴ The final amount of the investment is $5356.25

2. a) Determine the amount when $5000 is invested earning 7.32% annual interest over a 6 month period where interest is compounded monthly. [K: 2 marks]

$P = 5000$ $i = 0.0732$ per month

$n = 6$

$A = P(1+i)^n$

$= 5000(1+0.0732)^6$

$= 7639.31$

∴ The amount comes out to $7639.31 at the end of 6 months.

b) Determine the total interest earned.
[K: 1 mark]

$I = A - P$

$= 7639.31 - 5000$

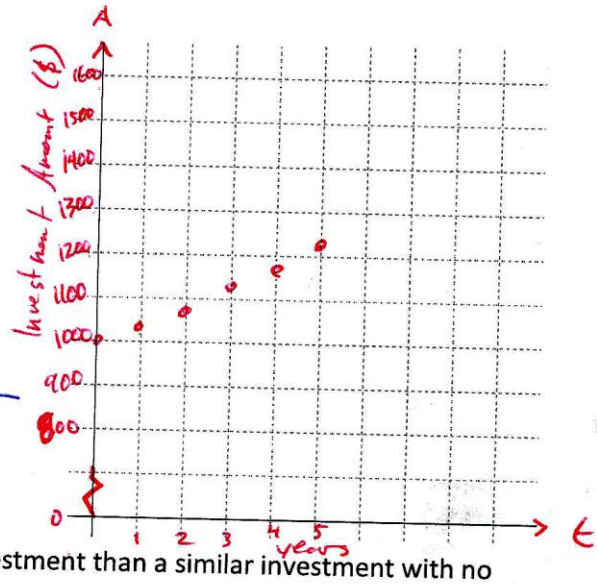
$= 2639.31$

∴ The interest earned is $2639.31

4. Gary invests $1000 in a 5-year investment that pays 4.1% interest per year, compounded annually.

a) Display the growth of Gary's investment using a table of values and a graph. [C: 8 marks]

Year	Principal ($)	Interest Earned ($)	Amount at End of Year ($)
1	1000	$1000 \times 0.041 \times 1 = 41.00$	$1000 + 41.00 = 1041.00$
2	1041.00	$1041.00 \times 0.041 \times 1 = 42.68$	$1041.00 + 42.68 = 1083.68$
3	1083.68	$1083.68 \times 0.041 \times 1 = 44.43$	$1083.68 + 44.43 = 1128.11$
4	1128.11	$1128.11 \times 0.041 \times 1 = 46.25$	$1128.11 + 46.25 = 1174.36$
5	1174.36	$1174.36 \times 0.041 \times 1 = 48.15$	$1174.36 + 48.15 = 1222.51$



b) How much more interest is earned in Gary's investment than a similar investment with no compounding? [A: 2 marks]

Gary earned $1222.51 in interest from a compounded interest investment

If he had gone with a simple interest investment he would earn in interest: $I = Prt = (1000)(0.041)(5) = 205$

Gary earns $17.51 more in the compounded interest investment.

5. Determine the amount if $2000 is invested at 5% per year, for 2 years at each compounding frequency. $P = 2000$

a) annually [K: 2 marks]

$n = 2\text{ years} \times \frac{1\text{ time}}{\text{year}} = 2\text{ times}$

$i = \frac{0.05/\text{year}}{1/\text{year}} = 0.05$

$A = P(1+i)^n$

$= 2000(1+0.05)^2$

$= 2205$

b) semi-annually [K: 2 marks]

$i = \frac{0.05/\text{year}}{2\text{ times/year}} = 0.025$

$n = 2\text{ years} \times \frac{2\text{ times}}{\text{year}} = 4$

$A = 2000(1+0.025)^4$

$= 2000(1.025)^4$

$= 2207.63$

c) quarterly [K: 2 marks]

$i = \frac{0.05/\text{year}}{4/\text{year}} = 0.0125$

$n = 2\text{ years} \times \frac{4\text{ times}}{\text{year}} = 8\text{ times}$

$A = 2000(1+0.0125)^8$

$= 2208.97$

d) monthly [K: 2 marks]

$i = \frac{0.05/\text{year}}{12\text{ months/year}} = 0.04167$

$n = 2\text{ years} \times 12\frac{\text{times}}{\text{year}} = 24\text{ times}$

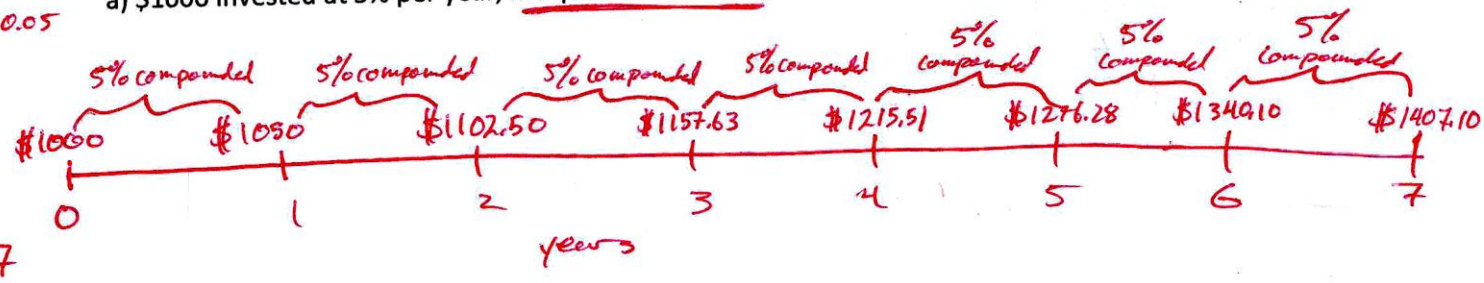
$A = 2000(1+0.04167)^{24}$

$= 2209.88$

6. Draw a time line to illustrate each situation.

a) $1000 invested at 5% per year, compounded annually for 7 years [C: 2 marks]

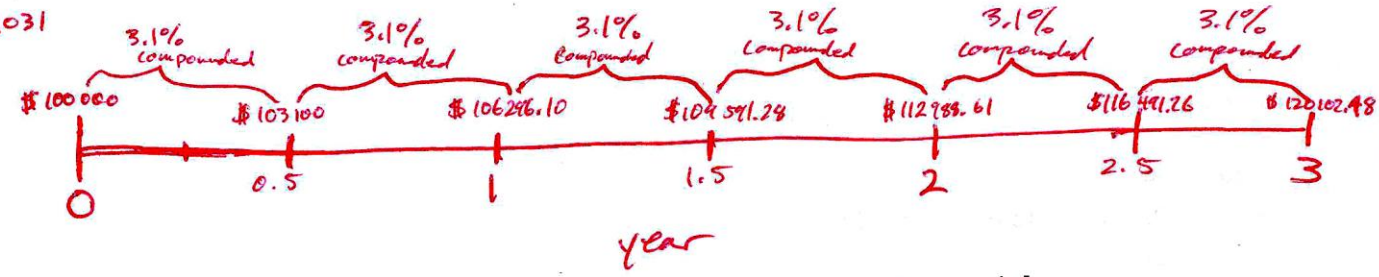
$i = 0.05$



$n = 7$

b) $100 000 invested at 6.2% per year, compounded semi-annually for 3 years [C: 2 marks]

$i = \frac{0.062}{2} = 0.031$

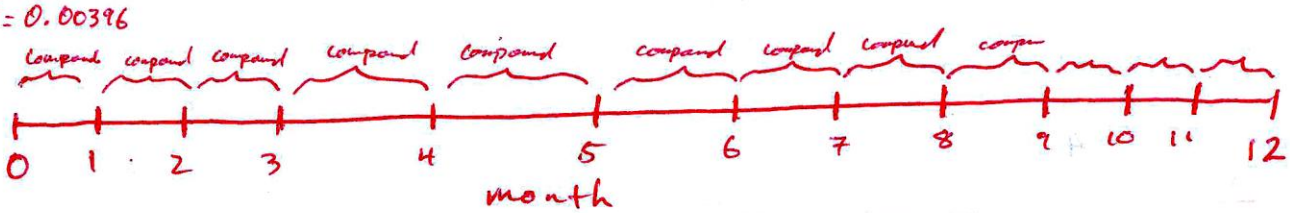


$n = 6$

c) $30 000 invested at 4.75% per year, compounded monthly for 1 year [C: 2 marks]

$i = \frac{0.0475}{12} = 0.00396$

$n = 12$



7. To purchase a car, Daniel borrows $2900 from his parents at 3.2% interest per year, compounded semi-annually. The loan must be paid in full at the end of 2 years.

a) How much money will Daniel need to repay his parents? [A: 2 marks]

$i = \frac{0.032/\text{year}}{2/\text{year}} = 0.016$

$n = 2\text{ years} \times \frac{2\text{ times}}{\text{year}} = 4\text{ times}$

$A = 2900(1+0.016)^4$
$= 3090.10$

∴ Daniel will need to repay his parents $3090.10 at the end of 2 years.

b) How much interest, in dollars, will Daniel pay? [A: 2 marks]

$I = A - P$
$= \$3090.10 - \$2900$
$= \$190.10$

∴ Daniel paid his parents $3090.10 at the end of 2 years.

8. Emma invests $3000 at 3.75% annual interest, compounded monthly for four years, and $3500 earning 4% simple interest.

a) Which investment earns Emma more money? [T: 4 marks]

Compound Investment

$i = \frac{0.0375/\text{year}}{12/\text{year}} = 0.003125$

$n = 4\text{ years} \times \frac{12\text{ times}}{\text{year}} = 48\text{ times}$

$P = 3000$

$A = 3000(1+0.003125)^{48}$
$= 3484.69$

$I = A - P$
$= 3484.69 - 3000$
$= 484.69$

Simple Investment

$r = 0.04$
$t = 4$
$P = 3500$

$I = Prt$
$= 3500 \times 0.04(4)$
$= 560$

Compound Investment

$I = 3484.69 - 3000$
$= 484.69$

Simple Investment

$I = 3500 \times 0.04(4)$
$= 560$

∴ The Simple investment earns Emma more money

9. In order to expand her business, Madison wishes to borrow $20 000. She has found two options.

A: a loan at an annual interest rate of 9% compounded monthly

B: a one-year loan at 9.25% per year, with quarterly compounding. The loan must be paid in full at the end of the term.

a) Which loan should Madison accept?
[T: 4 marks]

A

$i = \frac{0.09}{12} = 0.0075$

$n = 1\text{ year} \times 12\frac{\text{times}}{\text{year}} = 12\text{ times}$

$P = 20\ 000$

$A = P(1+i)^n$
$= 20\ 000(1+0.0075)^{12}$
$= 21\ 876.14$

The interest for plan A is

$I = A - P$
$= 21\ 876.14 - 20\ 000$
$= 1\ 876.14$

B

$i = \frac{0.0925}{4} = 0.023125$

$n = 1\text{ year} \times \frac{4\text{ times}}{\text{year}} = 4\text{ times}$

$P = 20\ 000$

$A = P(1+i)^n$
$= 20\ 000(1+0.023125)^4$
$= 21\ 915.17$

The interest for plan B is

$I = A - P$
$= 21\ 915.17 - 20\ 000$
$= 1915.17$

∴ Madison should go with Plan A because she pays less interest.

10. Calculate the present value of each amount.

a) $500 is required in 2 years. The money can be invested at 6% per year, compounded monthly. [K: 2 marks]

$A = 500 \quad n = 2 \times 12 = 24 \quad i = \frac{0.06}{12} = 0.005$

$P = \frac{A}{(1+i)^n} = \frac{500}{(1+0.005)^{24}}$

$P = 443.59$

b) $1000 is required in 6 months. The money can be invested at 4.35% per year, compounded semi-annually. [K: 2 marks]

$A = 1000 \quad i = \frac{0.0435}{2} = 0.02175$

$n = 1$ (because 6 months is half a year, and semi-annual is half a year)

$P = \frac{1000}{(1+0.02175)^1} = 978.71$

c) $800 is required in 1 year. The money can be invested at 3.75% per year, compounded quarterly. [K: 2 marks]

$A = 800 \quad i = \frac{0.0375}{4} = 0.009375$

$n = 1\text{ year} \times \frac{4\text{ times}}{\text{year}} = 4\text{ times}$

$P = \frac{800}{(1+0.009375)^4} = 770.69$

d) $3000 is required in 4 years. The money can be invested at 5.1% per year, compounded annually. [K: 2 marks]

$A = 3000 \quad i = \frac{0.051}{1} = 0.051$

$n = 4\text{ years} \times \frac{1\text{ time}}{\text{year}} = 4\text{ times}$

$P = \frac{3000}{(1+0.051)^4} = 2458.73$

11. Yolanda needs $18 000 in 4 years to buy a new car.

a) How much must she invest now in a GIC paying 5% per year, compounded annually, to have the required amount? [A: 3 marks]

$i = \frac{0.05}{1} = 0.05 \quad n = 4\text{ years} \times \frac{1\text{ time}}{\text{year}} = 4\text{ times} \quad A = 18\,000$

$P = \frac{18\,000}{(1+0.05)^4} = 14\,808.64$

∴ Yolanda needs to invest $14 808.64 in a GIC to buy a new car.

b) If the GIC in part a) has interest compounded monthly, how much less will Yolanda need to invest now? [A: 3 marks]

$i = \frac{0.05}{12} = 0.004167 \quad n = 4\text{ years} \times \frac{12\text{ times}}{\text{year}} = 48\text{ times} \quad A = 18\,000$

$P = \frac{18\,000}{(1+0.004167)^{48}} = 14\,743.04$

$\Delta P = 14\,808.64 - 14\,743.04 = 65.60$

∴ Yolanda would pay $65.60 less.

12. Calculate the discounted payment of each loan today.

a) A $200 debt is due in 3 months, discounted at an interest rate of 3% per year, compounded monthly. [K: 2 marks]

$i = \frac{0.03}{12} = 0.0025 \quad A = 200 \quad n = 3$

$P = \frac{A}{(1+i)^n}$ or $A(1+i)^{-n}$

$= \frac{200}{(1+0.0025)^3}$ or $200(1+0.0025)^{-3}$

$= 198.50$

b) A $450 debt is due in 1 year. Interest is 4.6% annually and compounded quarterly. [K: 2 marks]

$i = \frac{0.046}{4} = 0.0115 \quad A = 450 \quad n = 1\text{ year} \times \frac{4\text{ times}}{\text{year}}$

so $n = 4$ times

$P = \frac{450}{(1+0.0115)^4}$ or $450(1+0.0115)^{-4}$

$= 429.88$

c) A debt of $2000, due in 18 months is discounted at an interest rate of 6.62% per year, compounded semi-annually. [K: 2 marks]

$i = \frac{0.062}{2} = 0.031 \quad A = 2000$

$n = 18\text{ months} \times \frac{1\text{ time}}{6\text{ months}} = 3$ times

$P = \frac{2000}{(1+0.031)^3}$ or $2000(1+0.031)^{-3}$

$= 1824.96$

d) A debt of $5000, due in 2 years is discounted at 3.85% per year, compounded annually. [K: 2 marks]

$i = \frac{0.0385}{1} = 0.0385 \quad A = 5000$

$n = 2\text{ years} \times \frac{1\text{ time}}{\text{year}} = 2$ times

$P = \frac{5000}{(1+0.0385)^2}$ or $5000(1+0.0385)^{-2}$

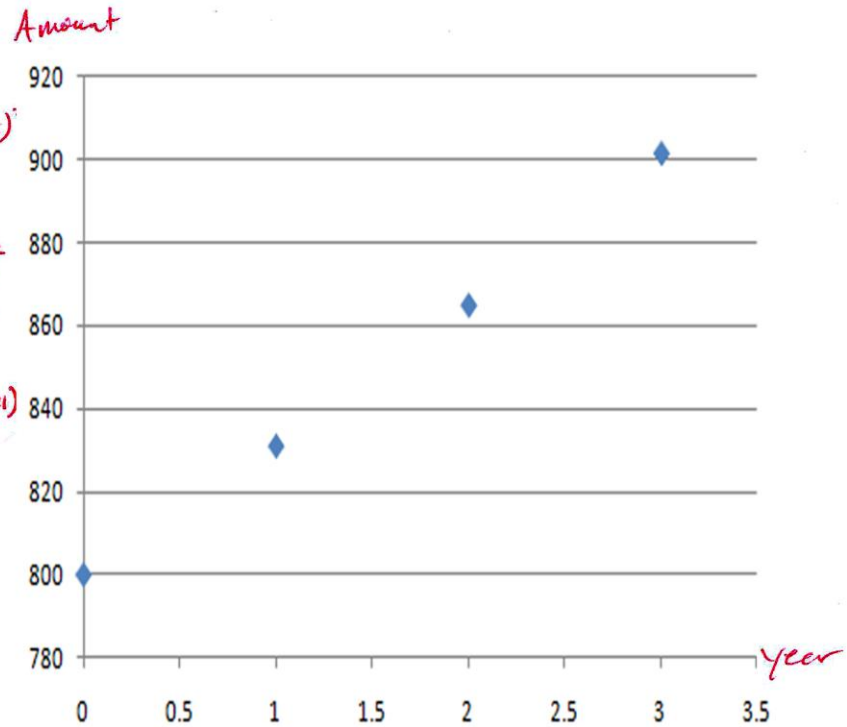
$= 4636.14$

14. Razeena decides to purchase an $800 GIC for 3 years with variable rates. Variable rates mean that the interest rate changes over the term of the GIC (Guaranteed Interest Certificate)

- In the 1st year, the GIC pays 3.85% annual interest.
- In the 2nd year, it pays 4.05% annual interest.
- In the 3rd year, it pays 4.2% annual interest.

Interest is compounded semi-annually. Display the growth of Razeena's investment using a table of values and a graph. [C: 3 marks]

Year	$A = P(1+i)^n$
1	$i = \frac{0.0385}{2} = 0.01925$ $P = 800$ $A = 800(1+0.01925)^1$ $A = 831.09$ $n = 1\text{year} \times 2\frac{\text{times}}{\text{year}} = 2\text{times}$
2	$i = \frac{0.0405}{2} = 0.02025$ $P = 831.09$ $n = 1\text{year} \times 2\frac{\text{times}}{\text{year}} = 2\text{times}$ $A = 831.09(1+0.02025)^2$ $= 865.09$
3	$i = \frac{0.042}{2} = 0.021$ $P = 865.09$ $n = 1\text{year} \times 2\frac{\text{times}}{\text{year}} = 2\text{times}$ $A = 865.09(1+0.021)$ $= 901.81$



15. Emma wants to have $7000 in 3 years to buy a motorcycle. His bank offers a 3-year GIC paying an annual interest rate of 4.32%, compounded annually.

How much would Emma need to invest today in order to have enough money to buy a motorcycle? [A: 3 marks]

$A = 7000$ $i = 0.0432$ $n = 3\text{ times} \times \frac{1\text{time}}{\text{year}} = 3\text{times}$

$$A = P(1+i)^n$$
$$P = A(1+i)^{-n}$$
$$P = 7000(1+0.0432)^{-3}$$
$$= 6165.88$$

∴ Emma needs to invest $6165.88 to have enough money to buy a motorcycle.

16. Madison needs to repay a $2500 loan in 1 year. How much might ~~Andrea's~~ Madison's creditor be willing to accept today at an annual interest rate of 6%, compounded monthly [A: 3 marks]

$i = \frac{0.06}{12} = 0.005$ $A = 2500$ $n = 1\text{ year} \times \frac{12\text{ times}}{\text{year}} = 12\text{ times}$

$P = 2500(1+0.005)^{-12}$
$= 2354.76$

∴ Madison's creditor would accept $2354.76

17. Daniel wants to have $1000 in 1 year to go on a vacation. His savings account pays an annual interest rate of 3.25%, compounded daily. How much money would Daniel need to have in the account today? [A: 3 marks]

$A = 1000$ $i = \frac{0.0325}{365} = 0.00008904 1$ $n = 1\text{ year} \times \frac{365\text{ times}}{\text{year}} = 365$

$P = 1000(1+0.000089041)^{-365}$
$= 968.02$

∴ Daniel would need to have $968.02 in the account today.

18. First-time homebuyers in Canada need to make a minimum down payment of 5% of the purchase price of a house. How much would first-time homebuyers need to invest today at 4.6% annual interest, compounded monthly, in order to have the minimum down payment on a $300 000 house in 5 years?
[T: 4 marks]

Cost of house is $300 000
So the down payment is $300\,000 \times 0.05 = 15\,000$

Thus $A = 15\,000$.

$i = \frac{0.046}{12} = 0.0038333$ $n = 5\text{ years} \times \frac{12\text{ times}}{\text{year}} = 60\text{ times}$

So $P = 15000(1+0.003833)^{-60}$
$= 11\,923.25$

∴ Canadian homebuyers would need to invest $11 923.25 to have enough for a down payment on a $300 000 house.