

K: /31

I: /19

C: /16

A: /18

**Multiple Choice** [K: 31 marks]

Identify the choice that best completes the statement or answers the question.

C 1 Express  $\sqrt[4]{1024}$  as a power with a base of 2.

a.  $2^{10}$

c.  $2^{\frac{5}{2}}$

b.  $2^{\frac{2}{5}}$

d.  $2^{\frac{3}{4}}$

$$\sqrt[4]{1024} = \sqrt[4]{2^{10}} = 2^{\frac{10}{4}} = 2^{\frac{5}{2}}$$

d 2 Express  $17^4$  as a power with a base of 2.

a.  $\log_2 17^4$

c.  $\log_2 17 \times \log_2 4$

b.  $2^{\frac{\log 2}{\log 17}}$

d.  $2^{\frac{4 \log 17}{\log 2}}$

$$2^x = 17^4$$

$$\log 2^x = \log 17^4$$

$$x \log 2 = 4 \log 17$$

$$x = \frac{4 \log 17}{\log 2}$$

$$\text{so } 2^{\frac{4 \log 17}{\log 2}} = 17^4$$

b 3 Express  $\frac{2^6 \times \left(\frac{1}{4}\right)^5}{\left(\sqrt[4]{16}\right)^3}$  as a power with a base of 4.

a.  $4^{-2}$

c.  $4^{\frac{3}{2}}$

b.  $4^{-\frac{7}{2}}$

d.  $4^{\frac{1}{2}}$

$$\frac{2^6 \cdot \left(\frac{1}{4}\right)^5}{\left(\sqrt[4]{16}\right)^3} = \frac{(2^3)^3 \cdot (4^{-1})^5}{(4\sqrt{4})^3}$$

$$= \frac{(4)^3 \cdot (4)^{-5}}{\left((4^{\frac{1}{2}})^2\right)^3}$$

$$= \frac{(4)^{-2}}{4^{\frac{3}{2}}}$$

$$= 4^{-2 - \frac{3}{2}} = 4^{-\frac{7}{2}}$$

C 4 Solve the equation  $4^{x-3} = 32^{2x+1}$ .

a.  $x = -1$

c.  $x = -\frac{11}{8}$

b.  $x = -\frac{1}{2}$

d.  $x = -2$

$$\rightarrow (2^2)^{x-3} = (2^5)^{2x+1} \rightarrow 2^{2x-6} = 2^{10x+5}$$

$$2x-6 = 10x+5$$

$$-5-6 = 10x-2x$$

$$-11 = 8x$$

$$x = -\frac{11}{8}$$

b 5 Solve the equation  $2 \times 16^x = 4 \times 8^{3x+2}$ .

a.  $x = -\frac{1}{2}$

c.  $x = 2$

b.  $x = -\frac{7}{5}$

d.  $x = -\frac{9}{5}$

$$\rightarrow 2(2^4)^x = 2^2(2^3)^{3x+2}$$

$$2(2^{4x}) = 2^2(2^{9x+6})$$

$$2^{4x+1} = 2^{9x+6+2}$$

$$4x+1 = 9x+8 \rightarrow -8+1 = 9x-4x$$

$$-7 = 5x$$

$$x = -\frac{7}{5}$$

- \* C 6 Solve the equation  $4 = 2.13^x$ . *use calculator*
- a.  $x \approx 1.87$        $x = \frac{\log 4}{\log 2.13} = 1.83$       c.  $x \approx 1.83$   
 b.  $x \approx 0.54$       d.  $x \approx 0.96$

$$4x+1 = 9x+8 \rightarrow -8+1 = 9x-4x$$

$$-7 = 5x$$

$$x = -\frac{7}{5}$$

- \* C 7 Solve the equation  $3 = 20\left(\frac{1}{2}\right)^{2x}$
- a.  $x \approx 5.47$       c.  $x \approx 1.37$   
 b.  $x \approx -0.52$       d.  $x \approx 2.31$

$$\log 3 = \log (20 \times (\frac{1}{2})^{2x}) \rightarrow \log 3 = \log 20 + \log (\frac{1}{2})^{2x}$$

$$\log 3 = \log 20 + 2x \log (\frac{1}{2})$$

$$\log 3 - \log 20 = 2x \log (\frac{1}{2})$$

$$x = \frac{\log 3 - \log 20}{2 \log (\frac{1}{2})} \text{ or } \frac{\log 3 - \log 20}{-2 \log (2)}$$

- \* d 8 Solve the equation  $24 = 6(3)^{3x+2}$
- a.  $x \approx 1.62$       c.  $x \approx -0.71$   
 b.  $x \approx 1.08$       d.  $x \approx -0.25$

$$\log 4 = (3x+2) \log 3 \rightarrow \log 4 = 3x \log 3 + 2 \log 3$$

- C 9 Evaluate, using the laws of logarithms.
- $\log_4 512 + \log_4 8 = \log_4 (2^9) + \log_4 (2^3)$
- a. 2      c. 6  
 b. 4      d. 8  
 d.  $\rightarrow 6 \log_4 4 = 6$

$$= \log_4 (2^9 \times 2^3)$$

$$= \log_4 (2^{12}) = \log_4 (4^6)$$

- a 10 Evaluate, using the laws of logarithms.
- $\log_4 128 - \log_4 32$
- a. 1  
 b. 2

$$= \log_4 (2^7) - \log_4 (2^5)$$

$$= \log_4 (\frac{2^7}{2^5})$$

$$= \log_4 (2^2)$$

$$= \log_4 4 = 1$$

- b 11 Evaluate, using the laws of logarithms.
- $\log_8 768 + \log_8 16 - \log_8 3$
- a. 2  
 b. 4

$$= \log_8 \left( \frac{768 \times 16}{3} \right) = \log_8 (256 \times 16)$$

$$= \log_8 (2^8 \times 2^4)$$

$$= \log_8 (2^{12}) = \log_8 (2^3)^4$$

$$= \log_8 (8)^4 = 4$$

- C 12 Evaluate, using the laws of logarithms.
- $\log_8 256 + \log_8 4 - \log_8 7 + \log_8 224$
- a. 3  
 b. 4

$$= \log_8 \left( \frac{256 \times 4 \times 224}{7} \right)$$

$$= \log_8 (256 \times 4 \times 32)$$

$$= \log_8 (2^8 \times 2^2 \times 2^5)$$

$$= \log_8 (2^{15}) = \log_8 (2^3)^5 = \log_8 (8)^5 = 5$$

a 13 Write as a single logarithm.

$$\log x^4 + \frac{1}{2} \log x^4$$

a.  $\log x^6$

b.  $\log x^8$

c.  $\log x^4 \sqrt{x}$

d.  $\frac{1}{2} \log x^8$

$$= \log x^4 + \log (x^4)^{\frac{1}{2}} = \log x^4 + \log x^2 = \log (x^4 \cdot x^2) = \log x^6$$

d 14 Write as a single logarithm.

$$\log \sqrt{x} - \log x^2 + \frac{3}{4} \log x^4$$

a.  $\log x$

b.  $2 \log x$

c.  $\frac{1}{2} \log x$

d.  $\frac{3}{2} \log x$

$$\begin{aligned} &= \log x^{\frac{1}{2}} - \log x^2 + \log (x^4)^{\frac{3}{4}} \\ &= \log x^{\frac{1}{2}} - \log x^2 + \log x^3 \\ &= \log \left( \frac{x^{\frac{1}{2}} \cdot x^3}{x^2} \right) = \log x^{\frac{1}{2}+3-2} \\ &= \log x^{\frac{3}{2}} \\ &= \frac{3}{2} \log x \end{aligned}$$

d 15 The restrictions on the variable in the expression  $\log(x^2 - 9) - \log(x^2 + 6x + 9)$  is

- a.  $x > 3$
- b.  $x < 3$

- c.  $x < -3$
- d. A and C

$$\begin{aligned} &x^2 - 9 > 0 \implies (x-3)(x+3) > 0 \\ &\implies x > 3, x < -3 \\ &x^2 + 6x + 9 > 0 \implies (x+3)(x+3) > 0 \\ &\implies (x+3)^2 > 0 \text{ Always} \end{aligned}$$

d 16 The restriction on the variable in the expression  $\log(2x^2 - 5x - 3) - \log(x^2 - 7x + 12)$  is

a.  $x > 4$

b.  $x > 3$

c.  $x < -\frac{1}{2}$

d. A and C

$$\begin{aligned} &2x^2 - 5x - 3 > 0 \\ &(2x+1)(x-3) > 0 \\ &\text{Scenario 1: Both +ve} \implies 2x+1 > 0 \implies x > -\frac{1}{2} \\ &\text{Scenario 2: Both -ve} \implies x-3 > 0 \implies x > 3 \end{aligned}$$

$$\begin{aligned} &x^2 - 7x + 12 > 0 \\ &(x-4)(x-3) > 0 \\ &\text{Scenario 1: both +ve} \implies x-4 > 0 \implies x > 4 \\ &\text{Scenario 2: both -ve} \implies x-4 < 0 \implies x < 4 \end{aligned}$$

$$\begin{aligned} &x^2 - 7x + 12 > 0 \\ &(x-4)(x-3) > 0 \\ &\text{Scenario 1: both +ve} \implies x-4 > 0 \implies x > 4 \\ &\text{Scenario 2: both -ve} \implies x-4 < 0 \implies x < 4 \end{aligned}$$

$$\begin{aligned} &x^2 - 7x + 12 > 0 \\ &(x-4)(x-3) > 0 \\ &\text{Scenario 1: both +ve} \implies x-4 > 0 \implies x > 4 \\ &\text{Scenario 2: both -ve} \implies x-4 < 0 \implies x < 4 \end{aligned}$$

b 17 Solve the equation  $\log_4 x = \log_4 15 - \log_4 3$ .

- a.  $x = 1.16$
- b.  $x = 5$

- c.  $x = 0.107$
- d.  $x = 4$

$$\begin{aligned} &\log_4 x = \log_4 \left( \frac{15}{3} \right) \\ &\text{Same base so...} \\ &x = \frac{15}{3} = 5 \end{aligned}$$

d 18 Solve the equation  $2 \log_4 64 = 3 \log_4 x$ .

- a.  $x = 4$
- b.  $x = 3$

- c.  $x = 1$
- d.  $x = 16$

$$\begin{aligned} &2 \log_4 64 = 3 \log_4 x \\ &2 \cdot 3 = 3 \log_4 x \\ &6 = 3 \log_4 x \\ &2 = \log_4 x \\ &x = 4^2 = 16 \end{aligned}$$

b 19 Solve the equation  $\frac{\log_6 46656}{\log_2 8} = \log_x 256$ .

- a.  $x = 2$
- b.  $x = 16$

- c.  $x = 12$
- d.  $x = 8$

$$\begin{aligned} &\log_x 256 = \frac{\log_6 (6^6)}{\log_2 (2^3)} = \frac{6 \log_6 6}{3 \log_2 2} \\ &= \frac{6 \cdot 1}{3 \cdot 1} = 2 \\ &\log_x 256 = 2 \\ &\text{so } x^2 = 256 \\ &x = \pm 16 \end{aligned}$$

a 20 Solve the equation  $\frac{\log_4 x}{\log_9 729} = 2 \log_4 \sqrt{16}$ .

a.  $x = 4096$   
 b.  $x = 2048$   
 c.  $x = 16$   
 d.  $x = 6$

*Handwritten solution:*  
 $\log_4 x = 2 \log_4 \sqrt{4^2}$   
 $\log_4 x = 2 \log_4 4$   
 $\log_4 x = 2$   
 $x = 4^2 = 16$

a 21 Solve the equation  $\log_3(2x+1) = 4$ .

a.  $x = 40$   
 b.  $x = \frac{11}{2}$   
 c.  $x = 20$   
 d.  $x = 7$

*Handwritten solution:*  
 $3^4 = 2x+1$   
 $81-1 = 2x$   
 $x = \frac{80}{2} = 40$

d 22 Solve the equation  $4 - \log_2(2-3x) = -1$ .

a.  $x = 4$   
 b.  $x = -2$   
 c.  $x = 2$   
 d.  $x = -10$

*Handwritten solution:*  
 $4+1 = \log_2(2-3x)$   
 $2^5 = 2-3x$   
 $32-2 = -3x$   
 $30 = -3x$   
 $x = -10$

a 23 Solve the equation  $\log(x-3)^2 = 2$ . \*  $x-3 > 0$  so  $x > 3$  \*

a.  $x = 13$   
 b.  $x = -7$   
 c.  $x = 3$   
 d. A and B

*Handwritten solution:*  
 $2 \log(x-3) = 2$   
 $\log(x-3) = 1$   
 $10^1 = x-3$   
 $x = 13$

*Method 2:*  
 $\sqrt{(x-3)^2} = \sqrt{10^2}$   
 $x-3 = \pm 10$   
 Case 1:  $x = 13$   
 Case 2:  $x = -7$  (rejected because  $x > 3$ )

c 24 Solve the equation  $\log_2(4-x)^2 = 4$ . \*  $4-x > 0$  so  $x < 4$  \*

a.  $x = -8$   
 b.  $x = 8$   
 c.  $x = 0$   
 d. B and C

*Handwritten solution:*  
 $2^4 = (4-x)^2$   
 $\sqrt{16} = \sqrt{(4-x)^2}$   
 $\pm 4 = 4-x$   
 Case 1 (+4):  $4 = 4-x$ ,  $0 = -x$ ,  $x = 0$   
 Case 2 (-4):  $-4 = 4-x$ ,  $-8 = -x$ ,  $x = 8$  (can't be Case 2 because  $x < 4$ )

c 25 Solve the equation  $\log_2 \sqrt{5x+1} = 4$ .

a.  $x = 3$   
 b.  $x = \frac{5}{3}$   
 c.  $x = 51$   
 d.  $x = 1$

*Handwritten solution:*  
 $\frac{1}{2} \log_2(5x+1) = 4$   
 $\log_2(5x+1) = 8$   
 $2^8 = 5x+1$   
 $256 = 5x+1$   
 $255 = 5x$   
 $x = 51$

*Handwritten note:*  
 $(x^2 + 24x) > 0 \rightarrow x(x+24) > 0$  so  $x < -24$  or  $x > 0$

b 26 Solve the equation  $\log_4 \sqrt{x^2 + 24x} = 2$ .

a.  $x = -32$   
 b.  $x = 8$   
 c. A and B  
 d.  $x = 4$

*Handwritten solution:*  
 $4^2 = x^2 + 24x$   
 $16 = x^2 + 24x$   
 $0 = x^2 + 24x - 16$   
 $0 = (x+32)(x-8)$   
 $x = -32$  or  $x = 8$  But  $x > -24$

a 27 Solve the equation  $\log_2 x + \log_2(x+1) = \log_2 6$ . \*  $x > 0$ ,  $x > -1$

a.  $x = 2$   
 b.  $x = -3$   
 c. neither A nor B  
 d. A and B

*Handwritten solution:*  
 $\log_2(x(x+1)) = \log_2 6$   
 $x^2 + x = 6$   
 $x^2 + x - 6 = 0$   
 $(x+3)(x-2) = 0$   
 $x = -3$  or  $x = 2$  But  $x > -1$

d 28 Solve the equation  $\log_6(x^2 + 19x + 124) = 3$ .   
 \*  $x^2 + 19x + 124 > 0$    
 no real roots. (ie) won't be negative.

a.  $x=4$   $6^3 = x^2 + 19x + 124$    
 b.  $x=-23$   $0 = x^2 + 19x + 124 - 216$    
 c. neither A nor B  $0 = x^2 + 19x - 92$    
 d. A and B  $0 = (x+23)(x-4)$    
 $x = -23 \quad x = 4$

b 29 Solve the equation  $\log(x+6) - \log(2x+1) = \log 4$ .

a.  $x=1$  \*  $x+6 > 0$   $2x+1 > 0$    
 ~~$x > 6$~~   ~~$x > -\frac{1}{2}$~~    
 b.  $x = \frac{2}{7}$   ~~$x > 6$~~   ~~$x > -\frac{1}{2}$~~    
 c.  $x = \frac{5}{7}$    
 d.  $x = \frac{9}{2}$

b 30 Solve the equation  $\log(3x^2 - 12) - \log(x+2) = \log 6$ .

a.  $x=0$  \*  $3x^2 - 12 > 0$   $x+2 > 0$    
 b.  $x=4$   $x^2 > 4$   $x > -2$    
 ~~$x < -2$~~   ~~$x > 2$~~    
 c.  $x=6$    
 d.  $x=1$

$\log\left(\frac{3x^2-12}{x+2}\right) = \log 6 \Rightarrow \frac{3x^2-12}{x+2} = 6$    
 $3x^2 - 12 = 6x + 12$    
 $x^2 - 4 = 2x + 4$    
 $0 = x^2 - 2x - 8$    
 $= (x-4)(x+2)$    
 $x = 4, -2$

b 31 Solve the equation  $\log_3(\log_2 x) = 1$ . \*  $x > 0$

a.  $x=2$   $3^1 = \log_2 x \Rightarrow 2^3 = x$    
 b.  $x=8$    
 c.  $x=4$    
 d.  $x=1$

$$32) (\sqrt{256})^{2x+3} = \left(\frac{1}{64}\right)^{x-5}$$

$$(\sqrt{2^8})^{2x+3} = ((64)^{-1})^{x-5}$$

$$\left(2^{\frac{8}{2}}\right)^{2x+3} = ((2^6)^{-1})^{x-5}$$

$$(2^4)^{2x+3} = (2^{-6})^{x-5}$$

$$2^{8x+12} = 2^{-6x+30}$$

$$8x+12 = -6x+30$$

$$8x+6x = 30-12$$

$$2x = 18$$

$$x = 9$$

$$33) 6^{3x+1} = 2^{2x-3}$$

$$\log 6^{3x+1} = \log 2^{2x-3}$$

$$(3x+1)(\log 6) = (2x-3)\log 2$$

$$3x \log 6 + \log 6 = 2x \log 2 - 3 \log 2$$

$$3x \log 6 - 2x \log 2 = -3 \log 2 - \log 6$$

$$x(3 \log 6 - 2 \log 2) = -3 \log 2 - \log 6$$

$$x = \frac{-3 \log 2 - \log 6}{3 \log 6 - 2 \log 2}$$

34)

$$\log_{18} 9 + \log_{18} 864 - \log_{18} 4 + \log_{18} 3$$

$$= \log_{18} \left( \frac{9 \times 864 \times 3}{4} \right) = \log_{18} (3^2 \times 3^6 \times 3)$$

$$= \log_{18} (3^2 \times 3^3 \times 8 \times 3)$$

$$= \log_{18} (3^6 \times 2^3)$$

$$= \log_{18} ((3^2)^3 \times 2^3)$$

$$= \log_{18} (9^3 \times 2^3)$$

$$= \log_{18} (18^3)$$

$$= \log_{18} (18)^3$$

$$= 3 \log_{18} (18)$$

$$= 3$$

$$36) \log \left( \frac{a^3 \sqrt{ab}}{(ab)^2} \right)$$

Method 1: simplify in bracket

$$\begin{aligned} & \log \left( \frac{a^3 \sqrt{ab}}{a^2 b^2} \right) \\ &= \log \left( \frac{a \sqrt{a} \sqrt{b}}{b^2} \right) \\ &= \log (a^{3/2} b^{1/2}) \\ &= \log a^{3/2} + \log b^{1/2} \\ &= \frac{3}{2} \log a + \frac{1}{2} \log b = \frac{3}{2} (\log a + \log b) \end{aligned}$$

Method 2: Use product & quotient law

$$\begin{aligned} & \log \left( \frac{a^3 \sqrt{ab}}{a^2 b^2} \right) \\ &= \log a^3 + \log \sqrt{a} + \log \sqrt{b} - \log a^2 - \log b^2 \\ &= 3 \log a + \frac{1}{2} \log a + \frac{1}{2} \log b - 2 \log a - 2 \log b \\ &= \log a (3 + \frac{1}{2} - 2) + \log b (\frac{1}{2} - 2) \\ &= \frac{3}{2} \log a - \frac{3}{2} \log b \\ &= \frac{3}{2} (\log a - \log b) \end{aligned}$$

$$37) \log_5 [\log_4 (\log_2 x)] = 1$$

$$5^1 = \log_4 (\log_2 x)$$

$$4^5 = \log_2 x$$

$$2^{4^5} = x$$

$$x = 2^{1024} = \text{some really big-ass number}$$

$$38) \log_{-2} \left( -\frac{1}{8} \right) = -3$$

Could be initially (and perhaps instinctively) written as

$$-2^{-3} = -\frac{1}{8} \quad \text{and although it makes sense algebraically}$$

it's not a valid logarithmic equation because

$$\log_a b = \frac{\log b}{\log a} \quad \text{and in this case we'd get}$$

$$\frac{\log \left( -\frac{1}{8} \right)}{\log (-2)} \quad \text{which is impossible because 10 raised to any exponent cannot equate a negative number.}$$

38)  $200\text{g} = A_0$     $t = 3\text{h}$     $A(3) = 140\text{g}$

a)  $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$

$140 = 200 \left(\frac{1}{2}\right)^{\frac{3}{h}}$

$\frac{140}{200} = \left(\frac{1}{2}\right)^{\frac{3}{h}}$

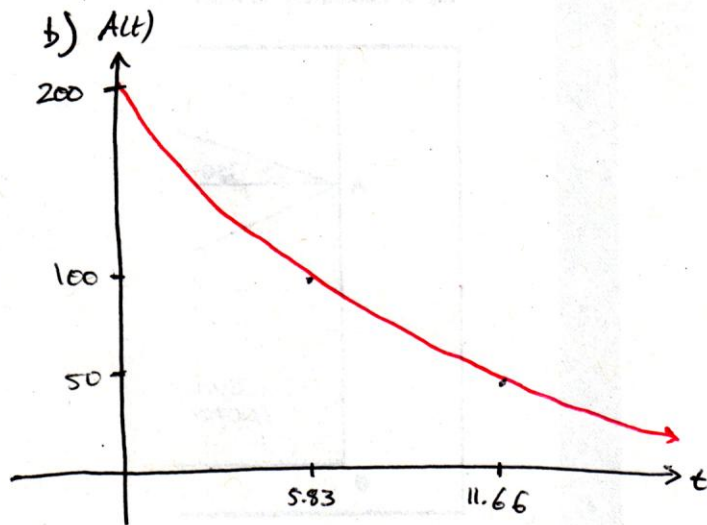
$\log\left(\frac{140}{200}\right) = \log\left(\frac{1}{2}\right)^{\frac{3}{h}}$

$\log\left(\frac{140}{200}\right) = \frac{3}{h} \log\left(\frac{1}{2}\right)$

$h = \frac{3 \log\left(\frac{1}{2}\right)}{\log\left(\frac{140}{200}\right)}$

$h = 5.83$

∴ the half-life of this substance is 5.83 hrs.



39)  $A_0 = 20\text{mg}$     $h = 3.1\text{min}$     $A(t) = 0.815 A_0$     $t = ? \text{seconds}$   
 $\frac{3.1 \times 60 \text{ s/min}}{186 \text{ s}}$

$0.815 A_0 = A_0 \left(\frac{1}{2}\right)^{\frac{t}{186}}$

$0.815(20) = 20 \left(\frac{1}{2}\right)^{\frac{t}{186}}$

$16.3 = 20 \left(\frac{1}{2}\right)^{\frac{t}{186}}$

$\frac{16.3}{20} = \left(\frac{1}{2}\right)^{\frac{t}{186}}$

$\log\left(\frac{16.3}{20}\right) = \log\left(\frac{1}{2}\right)^{\frac{t}{186}}$

$\log\left(\frac{16.3}{20}\right) = \frac{t}{186} \log\left(\frac{1}{2}\right)$

$t = \frac{186 \left(\log\left(\frac{16.3}{20}\right)\right)}{\log\left(\frac{1}{2}\right)}$

$t = 54.89 \text{ seconds}$

∴ It would take 54.89 seconds for polonium to reach 81.5% its initial mass.



40)  $h = 29$  years  $A(t) = ?$  after  $t = 18$  years if  $A_0 = 100$ g.

$$\begin{aligned} A(t) &= A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}} \\ &= 100 \left(\frac{1}{2}\right)^{\frac{18}{29}} \quad * \text{Use your calculator} \\ &= 100 (0.65036) \\ &= 65.036 \end{aligned}$$

$\therefore$  The remaining mass of strontium after 18 years is 65.04g, approximately.

41) 20% depreciation per year means it's 0.8 x original value.

$A(t) = 0.5A_0$  is what we want to find.

We want to find  $t$ ...

So... knowing...

$$A(t) = A_0 (0.8)^t$$

we have:  $0.5A_0 = A_0 (0.8)^t$

$$0.5 = (0.8)^t$$

$$\log(0.5) = \log(0.8)^t$$

$$\log(0.5) = t \log(0.8)$$

$$t = \frac{\log(0.5)}{\log(0.8)} = 3.106$$

$\therefore$  The time it takes for the car to depreciate to half its original value is 3.106 years, approximately.

41. A car depreciates at a rate of 20% per year. How long will it take for the car to reach half its original value? [A: 4 marks]

**Application Multiple Choice [A: 3 marks]**

- b 42. A 25-g sample of a substance decays to 13 g in 14 years. The half-life of this substance is approximately
- a. 3.77 years     Use  $A(t) = A_0 \left(\frac{1}{2}\right)^{t/h}$   
 b. 14.8 years      $13 = 25 \left(\frac{1}{2}\right)^{14/h}$   
 c. 8.59 years      $\frac{13}{25} = \left(\frac{1}{2}\right)^{14/h} \rightarrow \log \frac{13}{25} = \log \left(\frac{1}{2}\right)^{14/h}$   
 d. 14.1 years      $\log \frac{13}{25} = \frac{14}{h} \log \left(\frac{1}{2}\right) \rightarrow h = \frac{14 \log \left(\frac{1}{2}\right)}{\log \left(\frac{13}{25}\right)}$
- c 43. The half-life of a radioactive substance is 97 years. If 10 mg are produced after a reaction, approximately how long will it take for only 1 mg to remain?
- a. 29 years      $h = 97$  years  
 b. 180 years      $A_0 = 10$  mg      $A(t) = 1$  mg  
 c. 322 years      $1 = 10 \left(\frac{1}{2}\right)^{t/97}$   
 d. 462 years      $\frac{1}{10} = \left(\frac{1}{2}\right)^{t/97}$   
 $\log \left(\frac{1}{10}\right) = \log \left(\frac{1}{2}\right)^{t/97} \rightarrow \log \left(\frac{1}{10}\right) = \frac{t}{97} \log \left(\frac{1}{2}\right)$   
 $t = \frac{97 \log \left(\frac{1}{10}\right)}{\log \left(\frac{1}{2}\right)} = \frac{97 \log (10)}{\log (2)}$
- d 44. The value,  $D$ , in dollars, of a certain vehicle as a function of time,  $t$ , in years, depreciates according to the function  $D = 35\,000 \left(\frac{1}{2}\right)^{\frac{t}{4}}$ . How long will it take for this vehicle to be worth \$20 000?
- a. 0.80 years  
 b. 1.75 years  
 c. 4.26 years  
 d. 3.23 years
- $20\,000 = 35\,000 \left(\frac{1}{2}\right)^{\frac{t}{4}}$   
 $\frac{20\,000}{35\,000} = \left(\frac{1}{2}\right)^{\frac{t}{4}}$   
 $\log \left(\frac{20}{35}\right) = \log \left(\frac{1}{2}\right)^{\frac{t}{4}}$   
 $\log \left(\frac{4}{7}\right) = \frac{t}{4} \log \left(\frac{1}{2}\right)$   
 $t = \frac{4 \log \left(\frac{4}{7}\right)}{\log \left(\frac{1}{2}\right)}$

45. Choose a question from #33-49 that you are confident in answering correctly. You will teach how to solve this question in front of the entire class. [C: 10 marks]

[2 marks for correct solution, 2 marks for clarity in voice, 2 marks for clarity in writing, 2 marks for extemporaneous quality, 2 marks for paying attention to other classmates' presentation]