# MHF 4U1- Assignment Chapter 5A

**Multiple Choice**
*Identify the choice that best completes the statement or answers the question.*

a 1. For the function $y = A\sin\left(B(x - C)\right) - D$, which variable determines amplitude?

a. $A$ c. $C$
b. $B$ d. $D$

d 2. If a function is of the type $y = A\sin\left(B(x - C)\right) - D$ and the amplitude is 2, there is an expansion of 2, and a shift down of 3, what are the values of $A$, $B$, $C$ and $D$, respectively?

a. 0.5, 3, 0, 2 c. 0.5, 0, 2, 3
b. 2, 0.5, 3, 0 d. 2, 0.5, 0, 3

$D=3$ $A=2$ $B=\frac{1}{2}$

$C=0$ because no horiz. shift is listed.

a 3. A man programs his sprinkler system using the equation 15 sin ($kt$), where $t$ is in seconds. If he wants it to have a period of $\frac{6}{5}$ s, what should $k$ be?

a. $\frac{5\pi}{3}$ c. $\frac{5\pi}{6}$
b. $\frac{3\pi}{6}$ d. $\frac{6\pi}{3}$

$2\pi$ is standard period

New period $= \frac{2\pi}{k}$

$k = \frac{2\pi}{\frac{6}{5}} = \frac{10\pi}{6} = \frac{5\pi}{3}$

b 4. A Ferris wheel has a radius of 20 m, makes a full rotation in one minute and the axle stands 25 m above the ground. Which equation models the height of a chair of the Ferris wheel that starts at the top, where $x$ is in seconds?

a. $y = 20\sin\left(60x + \frac{\pi}{2}\right) + 25$ c. $y = 20\cos\left(\frac{\pi}{30}x - \frac{\pi}{2}\right) + 25$
b. $y = 20\sin\left(\frac{\pi}{30}x + \frac{\pi}{2}\right) + 25$ d. $y = 10\sin\left(\frac{\pi}{30}(x)\right) + 25$

means should be at max height when $x=0$

max height $= 20+25$
$= 45$m


$2\pi = 1$ minute

$2\pi = 60$s

$x \frac{2\pi}{60s} =$ amt of rad covered.

a 5. What is the period of the function $y = 10\sin\left(\frac{6\pi}{4}\left(x - \frac{\pi}{2}\right)\right) + 25$?

a. $\frac{4}{3}$ c. $\frac{6}{3}$
b. $\frac{4}{6}$ d. $\frac{3}{4}$

New period $= \frac{2\pi}{k}$

$= \frac{2\pi}{\frac{3\cancel{6}\pi}{4}}$

$= \frac{4}{3}$

d 6. Which of the following functions has the longest period?

a. $y = 6\sin(3x) + 20$
b. $y = 8\cos(2x) - 4$
c. $y = 7\cos(\pi x) + 13$
d. $y = 2\sin(0.5x) - 11$

The smaller the value of $k$, the longer the period because New Period $= \frac{2\pi}{k}$

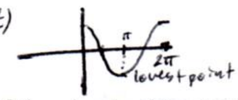
d 7. Which function is has a point closest to the origin?

a. $y = 2\sin(x) - 3$  $y(0) = -3$
b. $y = 2\cos(x) + 3$  $y(0) = 5$
c. $y = -2\sin(x) + 3$  $y(0) = 3$
d. $y = -2\cos(x) + 3$  $y(0) = 1$


$a = 6\text{cm}$

Lowest point is when $t = \pi$

$\cos(t)$




a 8. A pinwheel's axle stands 17 cm above ground. The edge of the pinwheel is at its lowest point $\pi$ seconds after it starts spinning and is 11 cm from the ground. What function best describes the height of the edge of the pinwheel?

a. $h(t) = 6\cos(t) + 17$
b. $h(t) = 6\cos(\pi t) + 17$
~~c.~~ $h(t) = \cancel{11}\cos(t) + 17$
~~d.~~ $h(t) = \cancel{11}\cos(\pi t) + 17$

b 9. A person stretching spins their arm around their shoulder once every 8 seconds. If the height of the person's shoulder is 2 m and their arm length is 1 m, which function models the height of the person's hand at time $t$, in seconds, if their hand starts at their side?

a. $\cos(t) + 2$
b. $-\cos\left(\frac{\pi t}{4}\right) + 2$
c. $-\cos\left(\frac{\pi t}{2}\right) + 2$
d. $\cos\left(\frac{\pi t}{4}\right) + 2$

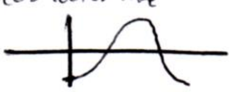
$a = 1\text{m}$

$P = 8s = \frac{2\pi}{k}$

$k = \frac{2\pi}{8s} = \frac{\pi}{4}$


Starts at lowest point at $t = 0$

cos normally has highest point at $t=0$. But, if we flipped by allowing $a = -1$ cos looks like



b 10. If $k = \frac{2\pi}{45}$, what is the period?

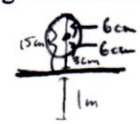
a. $\frac{45}{2}$
b. 45
c. $\frac{45}{2\pi}$
d. $\frac{2\pi}{45}$

$P = \frac{2\pi}{k} = \frac{2\pi}{\frac{2\pi}{45}} = 45$

c 11. A hamster wheel is in a cage on top of a table. If the high point of the wheel is 15 cm above the table and the lowest is 3 cm above the table and the table is 1 m off the ground, how high is the axis of the wheel relative to the ground?

top

a. 9 cm
b. 1.06 m
c. 1.09 m
d. 1.12 m




$\frac{15-3}{2} = \frac{12}{2} = 6\text{cm}$ off table
+ 3cm
+ 1m off ground
1.09m

a 12. In the equation $y = a\cos(k(x - d)) + c$, which constant determines the amplitude of the function?

a. $a$ c. $d$
b. $c$ d. $k$

a 13 The temperature of a swimming pool is cyclic and modelled by a trigonometric function. If its highest temperature is 82 °F and its lowest temperature is 76 °F, and it takes 12 hours for the temperature to change between its extremes, what equation models the temperature of the pool as a function of time in hours?

a. $y = 3\cos\left(\frac{2\pi}{24}t\right) + 79$ c. $y = 3\sin\left(\frac{2\pi}{24}t\right) + 79$

b. $y = 3\cos\left(\frac{2\pi}{12}t\right) + 79$ d. $y = 6\cos\left(\frac{2\pi}{24}t\right) + 79$

$\frac{82-76}{2} = \frac{6}{2} = 3$ = amplitude

$24 = \frac{2\pi}{k}$ so $k = \frac{2\pi}{24} = \frac{\pi}{12}$

Period = 24 hours $Y_{Max} = 82°F$ $Y_{min} = 76°F$

Because 12 hours from max to min

But period needs full revolution: max to min to max.

b 14 The productivity of a person at work (on a scale of 0 to 10) is modelled by a cosine function: $5\cos\left(\frac{\pi}{2t}\right) + 5$, where $t$ is in hours. If the person starts work at $t = 0$, being 8:00 a.m., at what times is the worker the least productive?

a. 12 noon c. 11 a.m. and 3 p.m.
b. 10 a.m. and 2 p.m. d. 10 a.m., 12 noon, and 2 p.m.

b 15 The height of a ball is modelled by the equation $h(t) = 4\sin(8\pi t) + 6.5$ where $h(t)$ is in metres and $t$ is in seconds. What are the highest and lowest points the ball reaches?

a. 10.5 m and 6.5 m c. 6.5 m and 2.5 m
b. 10.5 m and 2.5 m d. 14.5 m and 6.5 m

6.5 is middle
so max = 6.5 + 4 = 10.5
min = 6.5 − 4 = 2.5

a 16 What value for the function $y = 3\cos(t - \pi) + 2$ gives an instantaneous rate of change of 0?

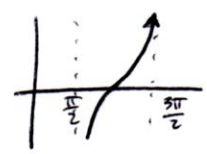
a. 0 c. $\frac{\pi}{3}$

b. $\frac{\pi}{2}$ d. $\frac{\pi}{4}$


But for $3\cos(t - \pi) + 2$
Graph shifts to the right by $\pi$
still would have $-\pi, 0, \pi, 2\pi \ldots$ to give rates of change as zero.

a 17 Determine the value of the average rate of change for $y = \tan(x)$ where $\frac{\pi}{2} < x < \frac{3\pi}{2}$

a. positive c. 0
b. negative d. undefined



d 18 A plane makes a loop in the air modelled by the function $h(t) = 3\cos\left(\frac{\pi}{16}t\right) + 5$, where $h$ is in km and $t$ is in seconds. If the plane makes only one full loop, what time(s) are the instantaneous rate of change 0?

a. 8, 24 c. 0, 32
b. 16, 48 d. 0, 16, 32

$\cos\theta$


when $\theta = 0, \pi, 2\pi \ldots$

so $\frac{\pi}{16}t = 0$ when $t = 0$
$\frac{\pi}{16}t = \pi$ when $t = 16$
$\frac{\pi}{16}t = 2\pi$ when $t = 32$

a 1. Which value for $x$ is a solution to $\cos x = \frac{\sqrt{2}}{2}$ ?

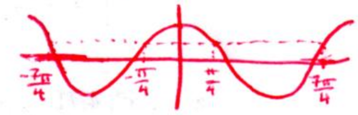
$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ so $x = \pm\frac{\pi}{4}, \pm\frac{7\pi}{4},$

a. $-\frac{9\pi}{4}$ $\cos\left(-\frac{9\pi}{4}\right) = \cos\left(-\frac{9\pi}{4}+2\pi\right)$ $= \cos\left(-\frac{\pi}{4}\right)$

b. $\frac{3\pi}{4}$

c. $-\frac{3\pi}{4}$

d. $\frac{4\pi}{3}$


d 2. Which value for $x$ is NOT a solution for $\tan x = 0$?

a. $2\pi$

b. 0

c. $-3\pi$

d. $\frac{\pi}{2}$

$\tan\theta$


$\tan\theta = 0$ when $\theta = 0, \pi, 2\pi \ldots$

c 3. What quadrants do the solutions to $2\sin x + \sqrt{3} = 0$ lie in?

a. I, IV

b. I, III

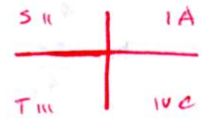
c. III, IV

d. I, II

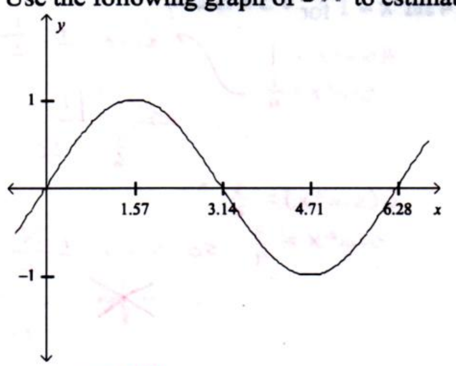
$2\sin x + \sqrt{3} = 0$

$2\sin x = -\sqrt{3}$

$\sin x = -\frac{\sqrt{3}}{2}$




d 6. Use the following graph of $g(x)$ to estimate the solution of $g(x) = -1$ for $0 \le x \le \cancel{6.28}$ $2\pi$.




$g(x) = -1$ when $x = \frac{3\pi}{2}$

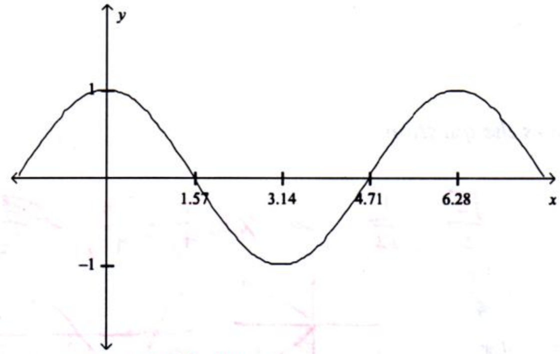
a. ~~1.57~~ $\frac{\pi}{2}$

b. ~~3.14~~ $\pi$

c. ~~−1.57~~ $-\frac{\pi}{2}$

d. ~~4.71~~ $\frac{3\pi}{2}$

a 22 Use the following graph of $f(x)$ to estimate the solution(s) of $f(x) = 0$ for $0 \le x \le 6.28$.




a. ~~1.57, 4.71~~ $\frac{\pi}{2}, \frac{3\pi}{2}$
b. ~~3.14~~ $\pi$
c. ~~0, 6.28~~ $0, 2\pi$
d. ~~-1.57~~, $-\frac{\pi}{2}$

a 23 A linear trigonometric equation involving $\tan x$ has one solution of $\frac{3\pi}{4}$. Which other possible value for $x$ is a solution?

a. $\frac{7\pi}{4}$
b. $-\frac{3\pi}{4}$
c. $\frac{\pi}{4}$
d. $\frac{5\pi}{4}$

y = tan x


* tan repeats every $\pi$ so...
$\frac{3\pi}{4} + \pi = \frac{7\pi}{4}$

b 24 Factor the expression $81 - 16\sin^2\theta$.   $a^2 - x^2$

a. $3(27 - 5\sin^2\theta)$
b. $(9 + 4\sin\theta)(9 - 4\sin\theta)$
c. $(9 - 4\sin\theta)^2$
d. $(3 - 2\sin^2\theta)^4$

we can factor
$81 - 16\sin^2\theta$
$= (9 - 4\sin\theta)(9 + 4\sin\theta)$

c 25 Factor the expression $\sin^2\theta - \sin\theta - 12$.   $x^2 - x - 12$

Let $x = \sin\theta$

a. $(\sin\theta - 3)(\sin\theta + 4)$
b. $(\sin\theta - 1)(\sin\theta + 12)$
c. $(\sin\theta + 3)(\sin\theta - 4)$
d. $(\sin\theta - 6)(\sin\theta + 2)$

we can factor
$(x-4)(x+3)$
$(\sin\theta - 4)(\sin\theta + 3)$

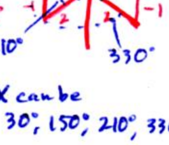
d 26 Which of the following is NOT a solution to the equation $4\sin^2 x = 1$ for $0 \le x \le 2\pi$?

a. 30°
b. 210°
c. 150°
d. 120°

$4\sin^2 x = 1$
$\sin^2 x = \frac{1}{4}$
$\sin x = \pm\frac{1}{2}$ $\frac{\text{opp}}{\text{hyp}}$




$\frac{\pi}{6} = 30°$




X can be 30°, 150°, 210°, 330°

a 27 Which is a solution for the equation $2(1-\cos^2 x) = \frac{3}{2}$?

a. 240°  c. 30°

b. 130°  d. 330°

$2(\sin^2 x) = \frac{3}{2}$

$\sin^2 x = \frac{3}{4}$ so $\sin x = \pm\frac{\sqrt{3}}{2}$

x can be $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

or 60°, 120°, 240°, 300°

a 28 Solve $(\sin x - 1)\left(\cos x - \frac{1}{2}\right) = 0$ where $0 \le x \le 2\pi$.

a. $\frac{\pi}{3}, \frac{\pi}{2}, \frac{5\pi}{3}$  c. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

b. $\frac{\pi}{6}, \frac{5\pi}{6}, \pi$  d. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

Just like finding roots, you can take each bracket:

$\sin x - 1 = 0$  and  $\cos x - \frac{1}{2} = 0$

$\sin x = 1$  $\cos x = \frac{1}{2}$

$x = \frac{\pi}{2},$  $x = \frac{\pi}{3}, \frac{5\pi}{3}$

$\tan\theta + 1 = 0$  $\cos\theta + 1 = 0$  $\sin\theta = 0$

$\tan\theta = -\frac{1}{1}$  $\cos\theta = -1$  $\theta = 0, \pi, 2\pi$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$  $\theta = \pi$

a 29 Which is NOT a solution to the equation $(\tan\theta + 1)(\cos\theta + 1)(\sin\theta) = 0$?

a. $\frac{\pi}{2}$  c. $\pi$

b. $\frac{3\pi}{4}$  d. $2\pi$

b 30 Which is a solution to the equation $(\sqrt{3}\tan\theta - 3)(8\cos x + 8) = 0$?

a. 360°  c. 330°

b. 60°  d. 90°

$\sqrt{3}\tan\theta = 3$  $8\cos x = -8$

$\tan\theta = \frac{3}{\sqrt{3}}$  $\cos x = -\frac{8}{8} = -1$

$\tan\theta = \frac{\sqrt{3}}{1}$  $x = \pi$ or 180°

$\theta = \frac{\pi}{3}$ or 60°

d 31 How many solutions does the equation $25\cos^2 x = 16$ have where $0 \le x \le 2\pi$?

a. 1  c. 3

b. 2  d. 4

$25\cos^2 x = 16$

$\cos^2 x = \frac{16}{25}$

$\cos x = \pm\frac{4}{5}$  $\frac{A}{H}$

32 $a = 4$ period $= 2\pi$ so $k = 1$ point $(0,2)$

$y = 4f(x) + c$ using $(0,2)$ we get

$2 = 4f(0) + c$ $f(x)$ is a cosine funtion as listed in the question

$2 = 4\cos(0) + c$

$2 = 4(1) + c$

$2 - 4 = c$

$c = -2$ So $y = 4\cos(x) - 2$

33 

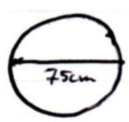

Let's say at $t=0$, second hand points straight upward. Thus height is at max at $2.04\,m$

Amplitude $= 0.04\,m$

Vertical shift is $2\,m$

Period is $60\,s = \frac{2\pi}{k}$ so $k = \frac{\pi}{30}$

With all that info we get $f(x) = 0.04\cos\left(\frac{\pi}{30}x\right) + 2$

34 


$\frac{1\,s}{7\,\text{rotations}} = \frac{2\pi}{k\,\text{rot}}$ so $k = 14\pi\ s^{-1}$

and $d = 75\,cm$ so $r = 37.5\,cm$ max height is $75\,cm$ and min height is $0\,cm$ Axle is at $37.5\,cm$


Amplitude $= 37.5\,cm$

$\therefore$ $y = 37.5\,cm\cos(14\pi t) + 37.5\,cm$

35 Looking at the function $h(t) = 15\cos\left(\frac{\pi}{3}t - 4\right) + \underbrace{18}_{\text{mid point}}$

we see 18 is the middle y-value.

We also see the amplitude as 15.

Thus the possible heights are $18 - 15 \leq h(t) \leq 18 + 15$

$3 \leq h(t) \leq 33$

36 Same procedure as #26

$$74-35 \leq h(t) \leq 74+35$$
$$39 \leq h(t) \leq 109$$

37 Fullmoon $\longrightarrow$ half moon — 13 days

we can assume that the same amount of time applies to change from half moon to no moon.

If we write out the sequence for a full period we see

Fullmoon $\xrightarrow[13\text{ days}]{}$ half moon $\xrightarrow[13\text{ days}]{}$ no moon $\xrightarrow[13\text{ days}]{}$ half moon $\xrightarrow[13\text{ days}]{}$ Full moon

52 days for a full period

so...

$$\frac{52\,\text{days}}{1\,\text{rev}} = \frac{2\pi}{k\ \text{rev}} \qquad k = \frac{2\pi}{52\,\text{days}}$$

$$k = \frac{\pi}{26}$$

38 $2\sin x - \cos^2 x = \sin^2 x$ for $0 \leq x \leq 2\pi$

$$2\sin x = \cos^2 x + \sin^2 x$$

*Use compound angle Trig identity*

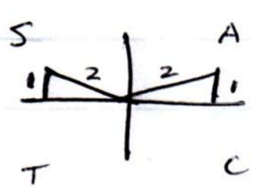
$$2\sin x = \cos(x)\cos(x) + \sin(x)\sin(x)$$

$$2\sin x = \cos(x - x)$$

$$2\sin(x) = \cos(0)$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2} \quad \frac{O}{H}$$




so

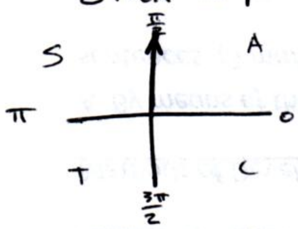
$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

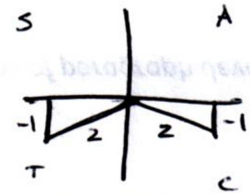
39 $$\sin^2 x - \frac{1}{2}\sin x - \frac{1}{2} = 0$$ where $0 \le x \le 2\pi$

* Factor *

$$(\sin x - 1)(\sin x + \frac{1}{2}) = 0$$

$\sin x = 1$ $\qquad$ $\sin x = -\frac{1}{2}$ $\quad \frac{O}{H}$




so

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

40 $$(\sin^2 x)(\sin 2x) = 0 \qquad 0 \le x \le 2\pi$$

$\sin^2 x = 0$ $\qquad$ $\sin 2x = 0$

when $x = 0, \pi, 2\pi$ $\qquad$ when $2x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi \ldots$

so...

$2x = 0 \rightarrow x = 0$

$2x = \pi \rightarrow x = \frac{\pi}{2}$

$2x = 2\pi \rightarrow x = \pi$

$2x = 3\pi \rightarrow x = \frac{3\pi}{2}$

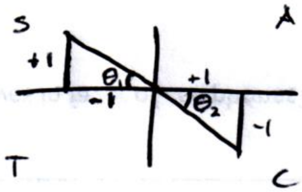
$2x = 4\pi \rightarrow x = 2\pi$

* you can stop here because our domain goes up to $0 \le x \le 2\pi$

So $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

41 $\tan(x) = -1 \qquad 0 \leq x \leq 2\pi$

$\tan(x) = \frac{-1}{1} \quad \frac{O}{A}$




$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

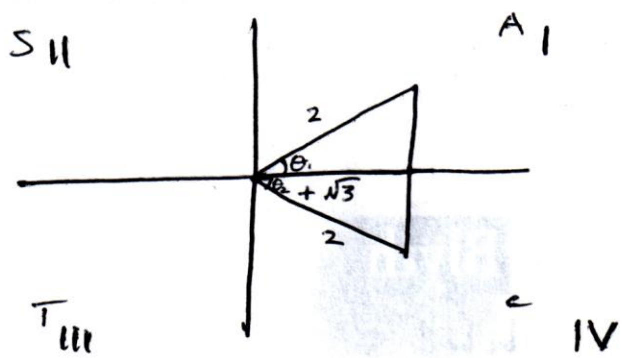
a) 2 solutions : $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

b) Quadrants II, IV

c) The related angle refers to $\theta_1$ and $\theta_2$ which is $\frac{\pi}{4}$

d) The actual solutions is $X = \frac{3\pi}{4}, \frac{7\pi}{4}$

42 $\cos x = \frac{\sqrt{3}}{2} \frac{A}{H} \quad 0 \leq x \leq 2\pi$




$X = \frac{\pi}{6}, \frac{11\pi}{6}$

a) 2 solutions are possible

b) Quadrants I, IV

c) Related angle is $\theta_1$ and $\theta_2$ which is $\frac{\pi}{6}$

d) Solutions are $X = \frac{\pi}{6}, \frac{11\pi}{6}$