MHF4U1-ASSIGNMENT CHAPTER 4 **NAME:____________________________**

K: /35 **I: /41** **C: /14** **A: /10**

Multiple Choice [K: 25 marks]

Identify the choice that best completes the statement or answers the question.

Try to avoid using a calculator for all the multiple choice questions. You won't have a calculator for the test, so answering these questions will be good practice to rely on your own rather than on a calculator.

d 1 The exact radian measure for an angle of 30° is

a. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$

b. $\frac{\pi}{3}$ d. $\frac{\pi}{6}$

$30° \times \frac{\pi}{180°} = \frac{\pi}{6}$

b 2 The exact radian measure for an angle of 135° is

a. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$

b. $\frac{3\pi}{4}$ d. π

$135° \times \frac{\pi}{180°} = \frac{3\pi}{4}$

a 3 Determine the exact degree measure for $\frac{2\pi}{5}$.

a. 72° c. 36°

b. 226° d. 144°

$\frac{2\cancel{\pi}}{5} \times \frac{180°}{\cancel{\pi}} = \frac{360°}{5} = 72°$

b 4 Determine the exact value of $\csc\frac{\pi}{4}$.

a. $\frac{1}{\sqrt{2}}$ c. $\frac{\sqrt{3}}{2}$

b. $\sqrt{2}$ d. $\frac{1}{2}$

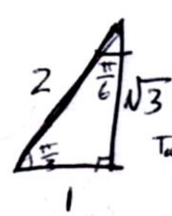
$\csc\frac{\pi}{4} = \frac{1}{\sin\frac{\pi}{4}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$\sin = \frac{O}{H}$

d 5 Determine the exact value of $\tan\frac{\pi}{6}$.

a. $\frac{2}{\sqrt{3}}$ c. $\frac{\sqrt{3}}{2}$

b. $\sqrt{3}$ d. $\frac{1}{\sqrt{3}}$



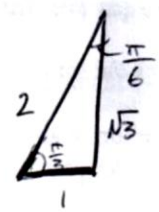
$\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\tan = \frac{O}{A}$

b 6 Determine the exact value of $\csc\frac{\pi}{6}$.

a. $\frac{2}{\sqrt{3}}$ c. $\frac{\sqrt{3}}{2}$

b. 2 d. $\frac{1}{2}$



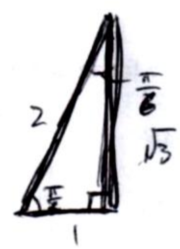
$\csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = \frac{1}{\frac{1}{2}}$

$= 2$

b 7 Determine the exact value of $\tan\frac{\pi}{3}$.

a. 2 c. $\frac{\sqrt{3}}{2}$

b. $\sqrt{3}$ d. $\frac{1}{\sqrt{3}}$



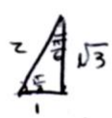
$\tan\frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}$

a 8 Determine the exact value of $\sec\frac{\pi}{3}$.

a. 2 c. $\frac{\sqrt{3}}{2}$

b. $\sqrt{3}$ d. $\frac{1}{2}$

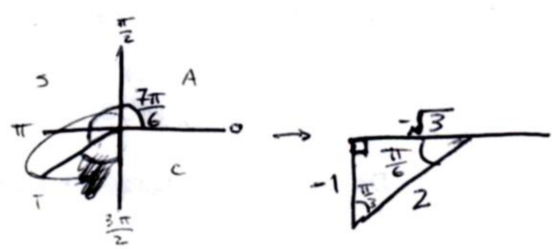
$\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$



c 9 Determine the exact value of $\cos\frac{7\pi}{6}$.

a. -2 c. $-\frac{\sqrt{3}}{2}$

b. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{\sqrt{3}}$

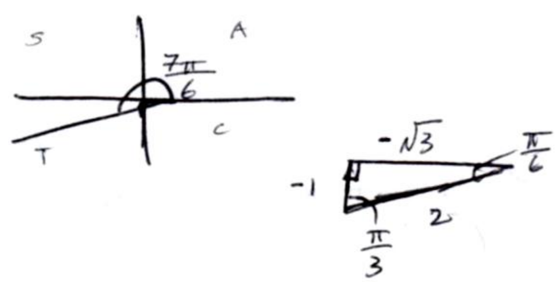


$\cos\frac{7\pi}{6} = \frac{-\sqrt{3}}{2}$

d 10 Determine the exact value of $\tan\frac{7\pi}{6}$.

a. $-\frac{1}{\sqrt{3}}$ c. $-\frac{\sqrt{3}}{2}$

b. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{\sqrt{3}}$



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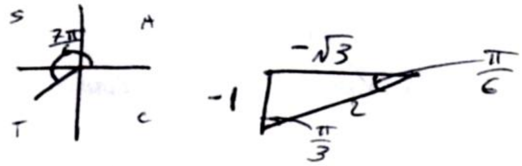
a 11 Determine the exact value of $\csc\frac{7\pi}{6}$.

a. -2 c. $-\frac{\sqrt{3}}{2}$

b. $\frac{\sqrt{3}}{2}$ d. $\frac{1}{\sqrt{3}}$

$\frac{1}{\sin\frac{7\pi}{6}}$

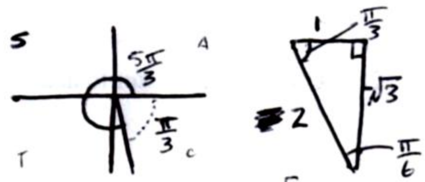


$\csc\frac{7\pi}{6} = \frac{1}{\sin\frac{7\pi}{6}} = \frac{1}{\frac{-1}{2}} = -2$

c 12 Determine the exact value of $\sin\frac{5\pi}{3}$.

a. -2 c. $-\frac{\sqrt{3}}{2}$

b. $\sqrt{3}$ d. $\frac{1}{2}$



$\sin\frac{5\pi}{3} = \frac{-\sqrt{3}}{2}$

d 13 Determine the exact value of $\sec\frac{5\pi}{3}$.

a. -2 c. $-\frac{\sqrt{3}}{2}$

b. $\sqrt{3}$ d. 2

$\frac{1}{\cos\frac{5\pi}{3}} =$ Use the same triangle as #12

$\frac{1}{\cos\frac{5\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

c 14 Determine the exact value of $\cot\frac{5\pi}{3}$.

a. $\frac{1}{\sqrt{3}}$ c. $-\frac{1}{\sqrt{3}}$

b. $\sqrt{3}$ d. $\frac{1}{2}$

$\frac{1}{\tan\frac{5\pi}{3}}$ Use the same triangle as #12

$\frac{1}{\tan\frac{5\pi}{3}} = \frac{1}{\frac{-\sqrt{3}}{1}} = \frac{1}{-\sqrt{3}}$

d 15 Determine the exact value of $\csc\pi$.

a. 0 c. 1

b. -1 d. undefined

$\csc\pi = \frac{1}{\sin\pi} = \frac{1}{0} =$ undefined

c 16 Determine the exact value of $\sec 2\pi$.

a. 0 c. 1

b. -1 d. undefined

$\sec 2\pi = \frac{1}{\cos(2\pi)} = \frac{1}{1} = 1$

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d ____ 17 the exact value of $\cot 2\pi$.

a. 0 c. 1

b. -1 d. undefined

$\cot 2\pi = \frac{1}{\tan 2\pi} = \frac{1}{0}$ = undefined

c ____ 18 Determine the exact value of $\csc \frac{\pi}{2}$.

$= \frac{1}{\sin\frac{\pi}{2}} = \frac{1}{1} = 1$

a. 0 c. 1

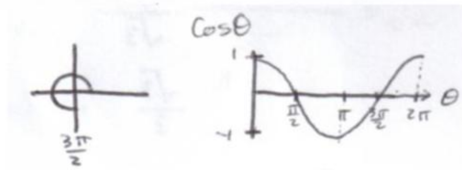
b. -1 d. undefined

d ____ 19 Determine the exact value of $\sec \frac{3\pi}{2}$.

$= \frac{1}{\cos\frac{3\pi}{2}} = \frac{1}{0}$ = undefined

a. 0 c. 1

b. -1 d. undefined



a ____ 20 An equivalent trigonometric expression for $\cos\left(\frac{\pi}{2} - x\right)$ is

a. $\sin x$ c. $\cos x$

b. $\tan x$ d. none of the above

c ____ 21 An equivalent trigonometric expression for $\sin\left(x + \frac{\pi}{2}\right)$ is

a. $\sin x$ c. $\cos x$

b. $\tan x$ d. none of the above

b ____ 22 An equivalent trigonometric expression for $\tan\left(x + \frac{\pi}{2}\right)$ is

a. $\tan x$ c. $\cot x$

b. $-\tan x$ d. none of the above

b ____ 23 An equivalent trigonometric expression for $\sin(-x)$ is

a. $\sin x$ c. $\cos x$

b. $-\sin x$ d. $-\cos x$

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a 24 Which compound angle expression is equivalent to $\sin x \cos y + \cos x \sin y$?

a. $\sin(x+y)$ c. $\sin(x-y)$

b. $\cos(x+y)$ d. $\cos(x-y)$

b 25 Which compound angle expression is equivalent to $\cos x \cos y - \sin x \sin y$?

a. $\sin(x+y)$ c. $\sin(x-y)$

b. $\cos(x+y)$ d. $\cos(x-y)$

Match each trigonometric expression with its equivalent expression. [K: 10 marks]

a. $\sin\left(\frac{\pi}{2}-x\right)$ c. $\cos\left(x+\frac{\pi}{2}\right)$ e. $\tan\left(\frac{\pi}{2}-x\right)$

b. $\cos\left(\frac{\pi}{2}-x\right)$ d. $\tan\left(x+\frac{\pi}{2}\right)$

a 26. $\cos x$ c 27. $-\sin x$ e 28. $\cot x$

b 29. $\sin x$ d 30. $-\cot x$

a. $2\sin x\cos x$ c. $1-\sin^2 x$ e. $\sec^2 x - 1$

b. $\dfrac{2\tan x}{1-\tan^2 x}$ d. $1-2\sin^2 x$

e 31. $\tan^2 x$ a 32. $\sin 2x$ b 33. $\tan 2x$

c 34. $\cos^2 x$ d 35. $\cos 2x$

Note: Just like the last assignment, question #36-47 involve short/long answer responses. Some of these questions may take up a whole page to write out (especially if you write out your answers out ***neatly***). Please write your answers in a ***neat*** and ***legible*** fashion so that not only can I mark it, but you will also be able to refer back to it as study material.
I will provide you paper, just like the last assignment. **FEEL FREE TO USE UP THE WHOLE PAGE** even if it's just for one question. There is no need to feel frugal and try to cram as many answers as possible in one page. Space out your answers. Just like before, if I see your solutions written on many of my paper in a ***neat*** fashion, I will give you a bonus mark of 1 in each achievement category (K/I/C/A) on this assignment. I hope this encourages both the reuse of old paper AND you write out your solutions ***clearly, neatly***, and ***legibly***.

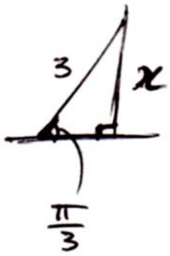
36. A 3-m ladder is leaning against a vertical wall such that the angle between the ground and the ladder is $\frac{\pi}{3}$.

What is the exact height that the ladder reaches up the wall? [A: 2 marks]
Draw a diagram. [C: 1 mark]

36. A 3-m ladder is leaning against a vertical wall such that the angle between the ground and the ladder is $\frac{\pi}{3}$.

What is the exact height that the ladder reaches up the wall? [A: 2 marks]

Draw a diagram. [C: 1 mark]



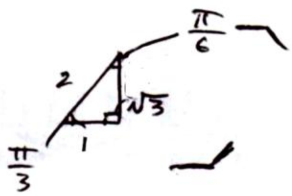
Because x lies opposite to the angle, $\frac{\pi}{3}$, and we know the hypotenuse is the length of the ladder (3m) we use

$$\sin\left(\frac{\pi}{3}\right) = \frac{x}{3}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{3}$$

$$x = \frac{3\sqrt{3}}{2}$$

Using special triangles we know $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

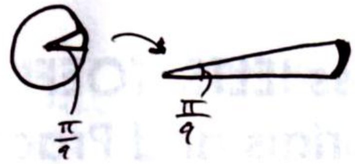


$\therefore$ the height that the ladder reaches is $\frac{3\sqrt{3}}{2}$ m.

37. A child swings on a playground swing set. If the length of the swing is 3 m and the child swings through an angle of $\frac{\pi}{9}$, what is the **exact** arc length through which the child travels? [A: 2 marks]

Draw a diagram. [C: 1 mark]

$l = 3m \quad \theta = \frac{\pi}{9}$



Arc Length

$$\theta = \frac{a}{r}$$

$$a = r\theta$$

$$a = 3m\left(\frac{\pi}{9}\right)$$

$$a = \frac{\pi}{3} m$$

$\therefore$ The child swings at/with an arc length of $\frac{\pi}{3}$ m.

38. A bicycle tire revolves at 150 rpm (revolutions per minute). What is its angular velocity, in radians per second, rounded to two decimal places? (*You'll need a calculator for this question) [A: 2 marks] Draw a diagram. [C: 1 mark]

$150 \frac{\text{revs}}{\text{min}}$ $\quad \omega = ? \frac{\text{rad}}{\text{s}}$ $\quad 1\text{ rev} = 2\pi \text{ rad}$

$\omega = \frac{150 \text{ revs}}{\text{min}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}}$ Unit conversion From revs to rad

$= \frac{300\pi \text{ rad}}{\cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ s}}$ Unit conversion from min to seconds

$= 5\pi \frac{\text{rad}}{\text{s}}$

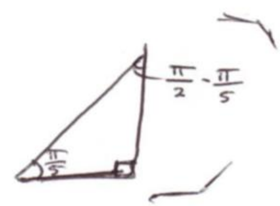
∴ the bicycle tire's angular velocity is $\frac{5\pi}{\text{s}}$ or $15.71 \frac{\text{rad}}{\text{s}}$

39. Given that $\sin x = \cos \frac{\pi}{5}$ and that x lies in the first quadrant, determine the exact measure of angle x.

[I: 2 marks]

$\sin x = \cos \frac{\pi}{5}$

* Knowing that $\cos \frac{\pi}{5}$ would look like this



We can use the trig cofunction where

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$

If $\frac{\pi}{2} - x = \frac{\pi}{5}$

then $x = \frac{\pi}{2} - \frac{\pi}{5}$

∴ $x = \frac{3\pi}{10}$

$x \cong 0.942$

40. Simplify $\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\frac{3\pi}{2} - x\right) + \sin(2\pi - x)$. [I: 10 marks]

$$\sin\left(\frac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\frac{3\pi}{2} - x\right) + \sin(2\pi - x)$$

Let's take each term and solve them 1 by 1 and put them all together at the end...

$$\begin{aligned}\sin\left(\tfrac{\pi}{2} - x\right) &= \sin\left(\tfrac{\pi}{2}\right)\cos x - \cos\left(\tfrac{\pi}{2}\right)\sin x \\ &= (1)\cos x - (0)\sin x \\ &= \cos x\end{aligned}$$

Use cofuntion
$\sin(x-y) = \sin x \cos y - \cos x \sin y$

Use the fact
$\sin\left(\frac{\pi}{2}\right) = 1$ and $\cos\left(\frac{\pi}{2}\right) = 0$

$$\begin{aligned}\sin(\pi - x) &= \sin(\pi)\cos x - \cos(\pi)\sin x \\ &= (0)\cos x - (-1)\sin x \\ &= \sin x\end{aligned}$$

Use cofunction

Use the fact
$\sin(\pi) = 0$ and $\cos(\pi) = -1$

$$\begin{aligned}\sin\left(\tfrac{3\pi}{2} - x\right) &= \sin\left(\tfrac{3\pi}{2}\right)\cos(x) - \cos\left(\tfrac{3\pi}{2}\right)\sin x \\ &= (-1)\cos(x) - (0)\sin x \\ &= -\cos(x)\end{aligned}$$

Use cofunction

Use the fact
$\sin\left(\frac{3\pi}{2}\right) = (-1)$ $\cos\left(\frac{3\pi}{2}\right) = 0$

$$\begin{aligned}\sin(2\pi - x) &= \sin(2\pi)\cos x - \cos(2\pi)\sin(x) \\ &= 0 \times \cos x - (1)\sin x \\ &= -\sin x\end{aligned}$$

So...

$$\begin{aligned}&\sin\left(\tfrac{\pi}{2} - x\right) + \sin(\pi - x) + \sin\left(\tfrac{3\pi}{2} - x\right) + \sin(2\pi - x) \\ &= \cos x + \sin x - \cos x - \sin x \\ &= \cos x - \cos x + \sin x - \sin x \\ &= 0\end{aligned}$$

41. Find the exact value of $\cos\frac{2\pi}{9}\cos\frac{\pi}{18}+\sin\frac{2\pi}{9}\sin\frac{\pi}{18}$. [I: 4 marks]

$$\underset{x}{\cos\left(\frac{2\pi}{9}\right)}\underset{y}{\cos\left(\frac{\pi}{18}\right)} + \underset{x}{\sin\left(\frac{2\pi}{9}\right)}\underset{y}{\sin\left(\frac{\pi}{18}\right)}$$

* there we see right away $\cos x \cos y + \sin x \sin y$
which is the cofunction $\cos(x-y)$

thus

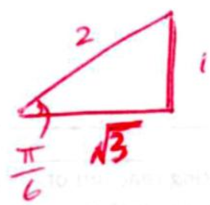
$$\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \sin\left(\frac{2\pi}{9}\right)\sin\left(\frac{\pi}{18}\right) = \cos\left(\frac{2\pi}{9}-\frac{\pi}{18}\right)$$

$$= \cos\left(\frac{3\pi}{18}\right)$$

$$= \cos\left(\frac{\pi}{6}\right)$$

Use special triangles

$$= \frac{\sqrt{3}}{2}$$



42. Find the exact value of $\dfrac{\tan\frac{\pi}{10}+\tan\frac{\pi}{15}}{1-\tan\frac{\pi}{10}\tan\frac{\pi}{15}}$. [I: 4 marks]

$$\frac{\tan\left(\frac{\pi}{10}\right)+\tan\left(\frac{\pi}{15}\right)}{1-\tan\left(\frac{\pi}{10}\right)\tan\left(\frac{\pi}{15}\right)}$$

where $x=\frac{\pi}{10}$ and $y=\frac{\pi}{15}$

$$= \tan(x+y)$$

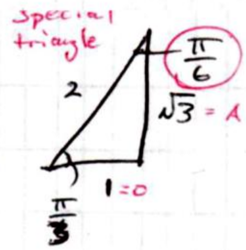
$$= \tan\left(\frac{\pi}{10}+\frac{\pi}{15}\right)$$

$$= \tan\left(\frac{3\pi}{30}+\frac{2\pi}{30}\right)$$

$$= \tan\left(\frac{5\pi}{30}\right)$$

$$= \tan\left(\frac{\pi}{6}\right)$$

$$= \frac{1}{\sqrt{3}}$$



$\mathrm{Tan}\,\theta = \frac{O}{A}$

43. Prove $1+\cos\theta = \dfrac{\sin^2\theta}{1-\cos\theta}$. [I: 3 marks]

$$1+\cos\theta = \frac{\sin^2\theta}{1-\cos\theta}$$

LS

$1+\cos\theta$

$= (1+\cos\theta)\times\left(\frac{1-\cos\theta}{1-\cos\theta}\right)$

$= \frac{1+\cos\theta-\cos\theta-\cos^2\theta}{1-\cos\theta}$

$= \frac{1-\cos^2\theta}{1-\cos\theta}$

$= \frac{\sin^2\theta}{1-\cos\theta}$

RS

$\frac{\sin^2\theta}{1-\cos\theta}$

LS = RS

44. Express $\sin(x+y+z)$ in terms of cosines and sines of x, y, and z. [I: 4 marks]

$$\sin(x+y+z)$$

$$\sin((x+y)+z) = \sin(x+y)\cos(z) + \cos(x+y)\sin(z)$$

$$= \left(\sin(x)\cos(y) + \cos(x)\sin(y)\right)\cos(z) + \left(\cos(x)\cos(y) - \sin(x)\sin(y)\right)\sin(z)$$

$$= \sin(x)\cos(y)\cos(z) + \cos(x)\sin(y)\cos(z) + \cos(x)\cos(y)\sin(z) - \sin(x)\sin(y)\sin(z)$$

45. If a and b are nonzero constants, simplify $\left\{\sqrt{3}a\sin\left(\frac{\pi}{b}\right)\right\}^2 + \left\{\sqrt{3}a\cos\left(\frac{\pi}{b}\right)\right\}^2$. [I: 4 marks]

$$\left\{\sqrt{3}\,a\sin\left(\frac{\pi}{b}\right)\right\}^2 + \left\{\sqrt{3}\,a\cos\left(\frac{\pi}{b}\right)\right\}^2$$

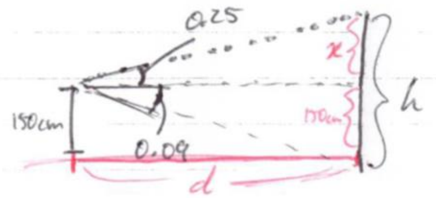
$$= 3a^2\sin^2\left(\frac{\pi}{b}\right) + 3a^2\cos^2\left(\frac{\pi}{b}\right)$$

$$= 3a^2\left(\sin^2\left(\frac{\pi}{b}\right) + \cos^2\left(\frac{\pi}{b}\right)\right)$$

$$= 3a^2(1)$$

$$= 3a^2$$

46. A 150-cm tall person stands on the bank of a narrow river and observes a flagpole on the opposite bank. If the angle of elevation to the top of the flagpole is 0.25 and the angle of depression to the bottom of the flagpole is 0.09, determine the flagpole's height, in metres, to two decimal places. [A: 4 marks]
Draw a diagram. [C: 1 mark]
(*You'll need a calculator for this question)



Calculate h by splitting the flagpole into 2. The bottom part is 150 cm
Let x represent top of flagpole.
Let d represent length of river.

① Find d first

$\tan(0.09) = \frac{150}{d}$

$d = \frac{150}{\tan(0.09)}$

② Use d to find x

$\tan(0.25) = \frac{x}{d}$

$x = d\tan(0.25)$

③ Add x to 150cm to find h.

$h = x + 150\text{cm}$

$=$

$\doteq 574\text{cm}$

or 5.74m

$\therefore$ The flagpole is 5.74m high.

47. Prove $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\tan\theta$ [I: 10 marks]

Left side Right side

$$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\tan\theta$$

$$\frac{1-(\cos^2\theta-\sin^2\theta)+\sin 2\theta}{1+\cos^2\theta-\sin^2\theta+\sin 2\theta}=\tan\theta$$

$$\frac{1-\cos^2\theta+\sin^2\theta+\sin 2\theta}{1-\sin^2\theta+\cos^2\theta+\sin 2\theta}=\tan\theta$$

$$\frac{\sin^2\theta+\sin^2\theta+\sin 2\theta}{\cos^2\theta+\cos^2\theta+\sin 2\theta}=\tan\theta$$

$$\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\tan\theta$$

$$\frac{\cancel{2}(\sin^2\theta+\sin\theta\cos\theta)}{\cancel{2}(\cos^2\theta+\sin\theta\cos\theta)}=\tan\theta$$

$$\frac{\sin\theta\,\cancel{(\sin\theta+\cos\theta)}}{\cos\theta\,\cancel{(\cos\theta+\sin\theta)}}=\tan\theta$$

$$\frac{\sin\theta}{\cos\theta}=\tan\theta$$

$$\tan\theta=\tan\theta$$

Left side equals right side