MHF4U1-ASSIGNMENT CHAPTER 2 **NAME:**_____

K: /60 **I:** /28 **C:** /16 **A:** /8

Multiple Choice [K: 20 marks]

Identify the choice that best completes the statement or answers the question.

d 1. Which of the following is the reciprocal of a linear function?

a. $f(x) = \frac{3}{x^2+1}$

b. $f(x) = \frac{1}{x^2+1}$

c. $f(x) = \frac{x}{x+3}$

d. $f(x) = \frac{1}{x+3}$

reciprocal: $x+3$ where $m=1$, $b=3$

b 2. Which of the following is the reciprocal of a quadratic function?

a. $f(x) = \frac{1}{4x+1}$

b. $f(x) = \frac{1}{x^2+1}$ — reciprocal — x^2+1

c. $f(x) = \frac{x}{x+1}$

d. none of the above

x^2, degree 2

d 3. Which of the following has a horizontal asymptote at $y = 0$?

a. $f(x) = \frac{1}{3-x}$ $\frac{\frac{1}{x}}{\frac{3}{x}-\frac{x}{x}} \to 0$ as $x \to \infty$

b. $f(x) = \frac{1}{17x+4}$ $\frac{\frac{1}{x}}{\frac{17x}{x}+4} \to 0$ as $x \to \infty$

c. $f(x) = -\frac{1}{x+2}$ $\frac{-\frac{1}{x}}{\frac{x}{x}+\frac{2}{x}} \to 0$ as $x \to \infty$

d. all of the above

b 4. What is true about the function $f(x) = \frac{1}{3x+5}$ as $x \to -\frac{5}{3}^{+}$? (from the right)

a. $f(x) \to 0$

b. $f(x) \to \infty$

c. $f(x) \to -\infty$

d. $f(x)$ is undefined

$f(-1) = \frac{1}{-3+5} = \frac{1}{2}$

$f(0) = \frac{1}{5}$

d 5. What is the x-intercept of $f(x) = \frac{1}{3x-4}$?

a. $-\frac{1}{4}$

b. $\frac{4}{3}$

c. $\frac{1}{4}$

d. There is no x-intercept.

$0 = \frac{1}{3x-4}$

Doesn't exist...

A horizontal asymptote exists instead.

a 6 What is the y-intercept of the function $f(x) = -\frac{3}{x-3} + 1$?

a. 2
b. –3
c. 1
d. 0

x = 0 so

$y = -\frac{3}{0-3} + 1$
$= -\frac{3}{-3} + 1$
$= 1 + 1$
$= 2$

a 7 What is the equation of the horizontal asymptote of $f(x) = -\frac{1}{2x+10}$?

a. $y = 0$
b. $y = 5$
c. $x = 0$
d. $x = 5$

$y = \frac{-\frac{1}{x}}{\frac{2x}{x} + \frac{10}{x}} = -\frac{\frac{1}{x}}{2 + \frac{10}{x}}$

As $x \to \infty$ $y \to 0$

so H. asymptote is $y = 0$

a 8 What is the value of k in the function $f(x) = \frac{3-k}{2x+k}$ if its graph passes through the point (5, –0.35) ?

a. 10
b. $-\frac{47}{6}$
c. $\frac{13}{4}$
d. No such k exists

$x = 5$ $y = -0.35$

$-0.35 = \frac{3-k}{2(5)+k} = \frac{3-k}{10+k}$

$(-0.35)(10+k) = 3-k$
$-3.5 - 0.35k = 3-k$
$k - 0.35k = 3 + 3.5$
$0.65k = 6.5$
$k = 10$

d 9 Which function is always positive?

a. $f(x) = \frac{3}{2x+4}$
b. $f(x) = \frac{1}{(x-4)^2}$
c. $f(x) = \frac{1}{x^2+4}$
d. B and C

– x can be any # because it's squared it's always positive.

x – 4 can be any number, but because it's squared, it's always +ve.

c 10 Which function has a y-intercept of $\frac{1}{2}$?

a. $f(x) = \frac{2}{(2x-1)(x+1)}$ $(0, \frac{1}{2})$ $= \frac{2}{(-1)(1)} = -2$
b. $f(x) = \frac{2}{2x^2+5x-3}$ $= \frac{2}{2(\frac{1}{2})^2 + 5(\frac{1}{2}) - 3} = \frac{2}{0}$
c. $f(x) = -\frac{4}{x^2-7x-8}$ $= -\frac{4}{0-0-8} = \frac{-4}{-8} = \frac{1}{2}$
d. all of the above

c 11 What is true about the function $f(x) = -\frac{1}{x^2+6x-7}$, as $x \to 1^-$?

a. $f(x) \to 0$ from below
b. $f(x) \to 0$ from above
c. $f(x) \to \infty$
d. $f(x) \to -\infty$

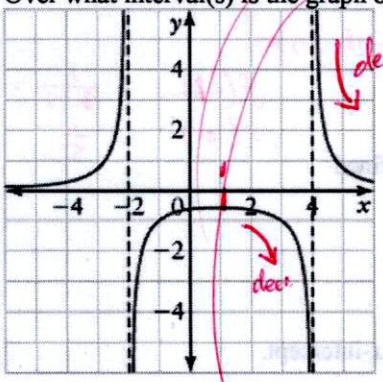
$f(x) = -\frac{1}{(x+7)(x-1)}$

as $x \to 1^-$ $= \frac{1}{(+ve)(-ve)}$

2 negatives makes a positive

so $f(x) \to \infty$ $+\infty$

c 12 Over what interval(s) is the graph of the rational function decreasing?



a. $-2 < x < 4$
b. $-2 < x, x \neq 4$
c. $x > 1, x \neq 4$
d. $x < -2, -2 < x < 1$

$(0, \frac{2}{3})$

d 13 Which function has a y-intercept of $\frac{2}{3}$?

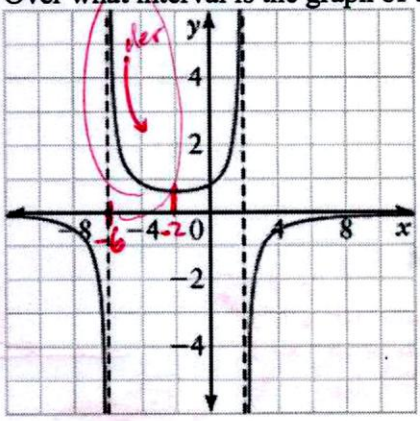
a. $f(x) = \frac{3x+2}{5x-1} = \frac{-0+2}{0-1} = +2$

b. $f(x) = \frac{3x+2}{5x-1} = \frac{2}{-1} = -2$

c. $f(x) = \frac{6x+5}{9x-7} = \frac{0+5}{0-7} = -\frac{5}{7}$

d. $f(x) = -\frac{5x+6}{7x-9} = -\frac{0+6}{0-9} = +\frac{2}{3}$ ✓

b 14 Over what interval is the graph of the rational function positive but decreasing?



b 15 Which function has a vertical asymptote at $x = 3$? Take the denominator and equal it to 0.

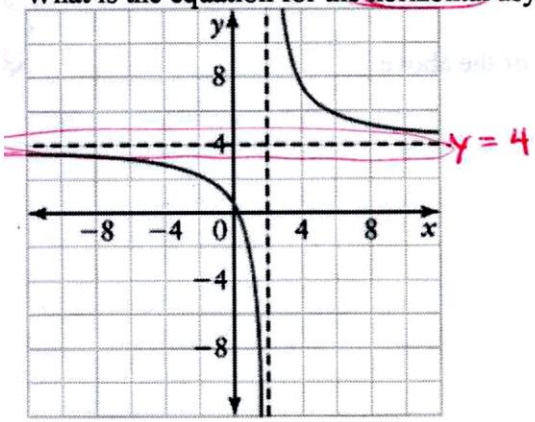
a. $f(x) = \frac{x+3}{5x-1}$ $5x-1=0$, $x=\frac{1}{5}$

b. $f(x) = \frac{x+3}{x-3}$ $x-3=0$, $x=3$

c. $f(x) = \frac{x-3}{x-1}$ $x-1=0$, $x=1$

d. $f(x) = \frac{3x-5}{x-1}$ $x-1=0$, $x=1$

d 16 What is the equation for the horizontal asymptote of the graph of the function shown?



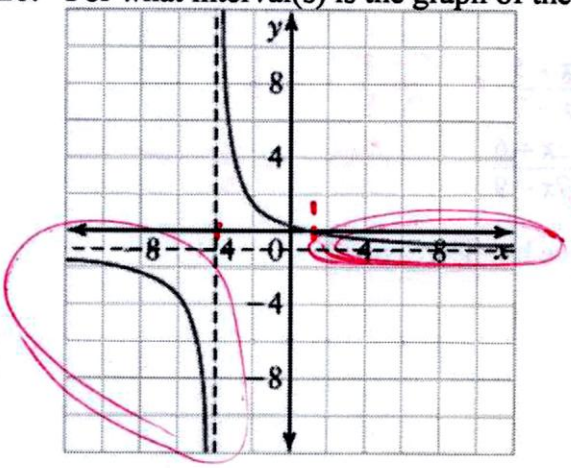
a. $x = 2$

b. $x = 4$

c. $y = 2$

d. $y = 4$

c 17 20. For what interval(s) is the graph of the rational function negative?



a. $x < -4$

b. $x > 1$

c. $x < -4, x > 1$

d. $x \in \mathbb{R}, x \neq -4$

d 18 Where is the function $f(x) = \dfrac{x-4}{2x-6}$ increasing?

a. $x \geq 4, x < 3$

b. $x > 3$

c. $x > 6$

d. $x \in \mathbb{R}, x \neq 3$

$2x-6=0$

Vert Asymptote at $x=3$

Choose values left of $x=3$

2) $f(0) = \frac{2}{3}$

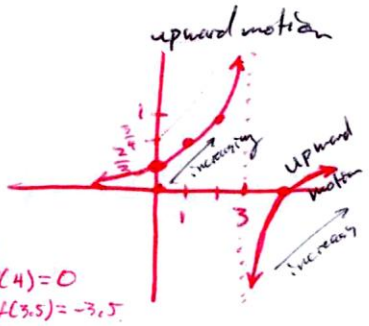
$f(1) = \frac{-3}{-4} = \frac{3}{4}$

$f(2) =$ 1

Choose values to the right of 3.

$f(4) = 0$

$f(3.5) = -3.5$



d 19 What is true about the function $f(x) = \dfrac{2x+5}{x+3}$ as $x \to \infty$?

a. ~~$f(x) \to \frac{5}{3}$ from above~~

b. ~~$f(x) \to \frac{5}{3}$ from below~~

$\dfrac{\frac{2x}{x} + \frac{5}{x}}{\frac{x}{x} + \frac{3}{x}} = \dfrac{2 + \frac{5}{x}}{1 + \frac{3}{x}}$ so as $x \to \infty$ $y \to 2$

c. $f(x) \to 2$ from above

d. $f(x) \to 2$ from below

$f(7) = \frac{14+5}{7+3} = \frac{19}{10}$

$f(17) = \frac{34+5}{17+3} = \frac{39}{20}$

increasing to 2 so ~~from~~ below

c 20 What is the value of k in the function $f(x) = \dfrac{2x+k}{x+3}$ if its graph passes through the point (2, 4.2)?

a. 21.2

b. 6

c. 17

d. none of the above

$(4.2) = \frac{2(2)+k}{2+3}$

$(4.2)(5) = 4+k$

$21.0 = 4+k$

$k = 21-4 = 17$

b 21 Solve the equation $\dfrac{1}{x-4} = \dfrac{5}{x}$.

$x \neq 4$ $x \neq 0$

$x = 5(x-4)$

$x = 5x-20$

$20 = 5x-x$

$20 = 4x$

$x = 5$

a. $x = -1$

b. $x = 5$

c. $x = -5$

d. no solution

d 22 Solve the equation $\dfrac{3}{x-2} = \dfrac{7}{4x-8}$.

$x \neq 2$

$3(4x-8) = 7(x-2)$

$12x-24 = 7x-14$

$12x-7x = -14+24$

$5x = 10$

$x = 2$ BUT!

a. $x = 2$

b. $x = \frac{4}{5}$

$x = -2$

no solution

b 23 What are the x-intercepts of the graph of $f(x) = \dfrac{x^2+4x-}{x^2-x-} = \dfrac{(x+7)(x-3)}{(x-5)(x+4)}$

$y=0$

$x \neq 5, -4$

a. $-4, 5$

b. $-7, 3$

c. $4, -5$

d. $7, -3$

a 24 For what values of k does the graph of $f(x) = \dfrac{x^2 + 16x + k}{x^2 + 7x}$ have no x-intercepts?

$f(x) = \dfrac{x^2+16x+k}{x(x+7)}$ $\quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

To have no x-intercepts $\quad b^2 - 4ac < 0$

$16^2 - 4(1)k < 0$

$16^2 < 4k$

$k > 64$

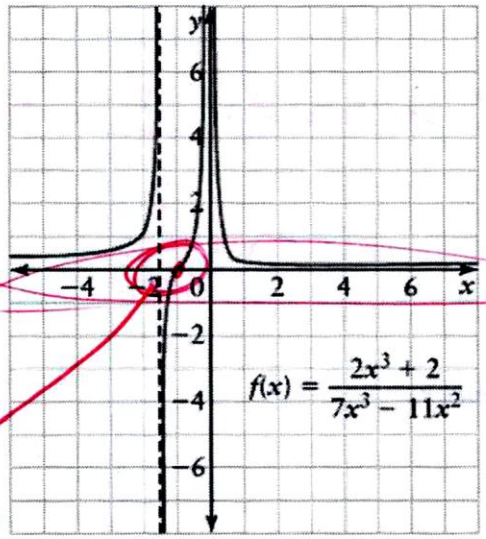
a. $k > 64$

b. $k < 64$

c. $k > 4$

d. $k < 4$

b 25 Use the graph of $f(x) = \dfrac{2x^3 + 2}{7x^3 + 11x^2}$ to solve the equation $0 = \dfrac{2x^3 + 2}{7x^3 + 11x^2}$.



root at $x = -1$

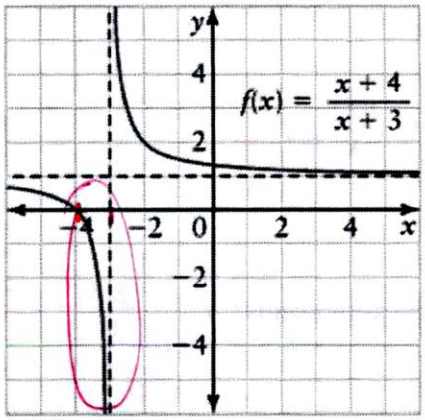
a. $x = -1.5$

b. $x = -1$

c. $x = 0$

d. no solution

C 26 Use the graph of $f(x) = \dfrac{x+4}{x+3}$ to solve the inequality $\dfrac{x+4}{x+3} \le 0$.



a. $x = -4$

b. $-4 < x < -3$

c. $-4 \le x < -3$

d. no solution

Should read $-4 \le x < -3$

Matching

Match each graph of a rational function with its equation.

a. $f(x) = \dfrac{10}{x^2+4}$

b. $f(x) = \dfrac{1}{x-4}$

c. $f(x) = -\dfrac{1}{x-4}$

d. $f(x) = \dfrac{1}{x^2-4}$

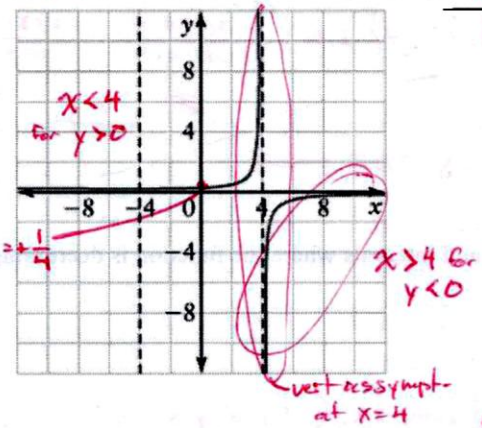
e. $f(x) = \dfrac{2x-10}{x-4}$

f. $f(x) = \dfrac{10}{x^2-2x-8}$

g. $f(x) = \dfrac{1}{(x-4)^2}$

h. $f(x) = \dfrac{x}{2(x-4)}$

____ 27.



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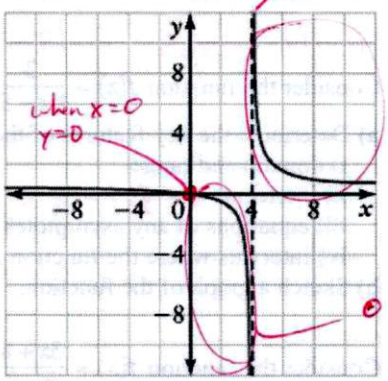

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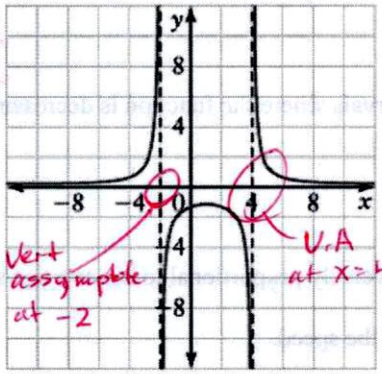

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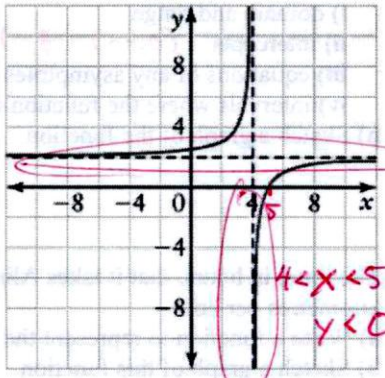

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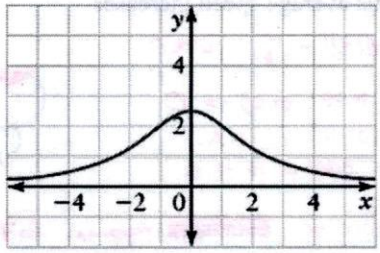


32)

a) $x \neq 4, 0$

$\frac{1}{x-4} = -\frac{6}{x}$

$x = -6(x-4)$

$x = -6x + 24$

$7x = 24$

$x = \frac{24}{7}$

$x \neq 3, 0$

b) $\frac{x+5}{x-3} = \frac{2x+7}{x}$

$x(x+5) = (2x+7)(x-3)$

$x^2 + 5x = 2x^2 - 6x + 7x - 21$

$0 = 2x^2 - 6x + 7x - 21 - x^2 - 5x$

$= 2x^2 - x^2 - 6x + 7x - 5x - 21$

$= x^2 - 4x - 21$

$= (x-7)(x+3)$

$x = 7, -3$

c) $x \neq 2$

$\frac{3}{2x-4} = \frac{4}{x-2}$

$3(x-2) = 4(2x-4)$

$3x - 6 = 8x - 16$

$16 - 6 = 8x - 3x$

$10 = 5x$

$x = 2$

No solution.

33) V.A at $x = -2$ for $f(x) = \frac{1}{kx - C}$ with $y_{int} = -\frac{1}{8}$

2 pieces of information

Use both to solve for k and C

VA at $x = -2$ and $y_{int} = -\frac{1}{8}$

VA is when $kx - C = 0$

$k(2) - C = 0$

$2k = C$

$2k = 8$

$k = 4$

$-\frac{1}{8} = \frac{1}{k(0) - C}$

$-\frac{1}{8} = \frac{1}{0 - C}$

$-\frac{1}{8} = \frac{1}{-C}$

$C = 8$

Use by plugging in

$\therefore f(x) = \frac{1}{4x - 8}$

34) $f(x) = \frac{3}{4x-5}$

a)

i) $\{x \in \mathbb{R},\ x \neq \frac{5}{4}\}$ found by looking at denominator

$\{y \in \mathbb{R},\ y \neq 0\}$ found by trying to find any H.A's $\frac{\frac{3}{x}}{4-\frac{5}{x}} \to 0$ as $x \to \infty$

ii) No x-intercepts because $y \neq 0$

$y_{int} = -\frac{3}{5}$ because $f(0) = \frac{3}{4(0)-5} = -\frac{3}{5}$

iii) V.A $\underbrace{4x-5=0}$. so $x = \frac{5}{4}$

Denominator of $f(x)$

H.A $\frac{\frac{3}{x}}{4-\frac{5}{x}} \to 0$ as $x \to \infty$

iv) Look at V.A. and test left and right of point(s)

Left of $x = \frac{5}{4}$

ie) $x < \frac{5}{4}$

slope of secant $\frac{f(1)-f(0)}{1-0} = \frac{-\frac{3}{1} - (-\frac{3}{5})}{1}$

$= -\frac{12}{5}$

∴ decreasing left of $\frac{5}{4}$

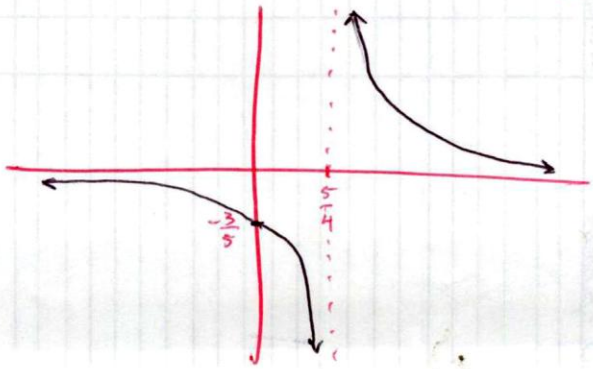
Right of $x = \frac{5}{4}$

ie) $x > \frac{5}{4}$

$\frac{f(3)-f(2)}{3-2} = \frac{\frac{3}{7} - 1}{1}$

$= -\frac{4}{7}$

∴ decreasing right of $\frac{5}{4}$



35) $f(x) = \frac{3x+8}{x-2}$

a) i) $\{x \in \mathbb{R}, x \neq 2\}$

$\{y \in \mathbb{R}, y \neq 3\}$

ii) x_{int} at $-\frac{8}{3}$ found by $0 = \frac{3x+8}{x-2}$

$0 = 3x+8$

$x = -\frac{8}{3}$

y_{int} at -4 found by $f(0) = \frac{3(0)+8}{0-2}$

$= \frac{0+8}{-2}$

$= -4$

iii) V.A $x-2=0$

so $x=2$

H.A. $\frac{\frac{3x}{x}+\frac{8}{x}}{\frac{x}{x}+\frac{2}{x}} = \frac{3+\frac{8}{x}}{1-\frac{2}{x}}$

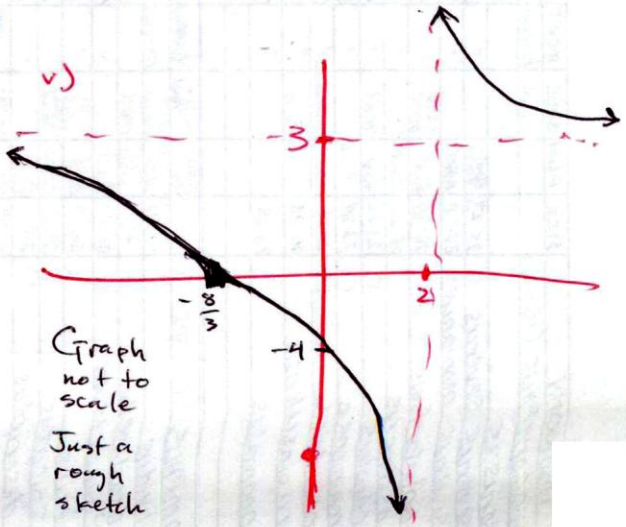
as $x \to \infty$ $\frac{3+\frac{8}{x}}{1-\frac{2}{x}} \to 3$

so $y=3$ is an H.A

iv) Look at V.A and find slopes of secants left and right of point(s)

Left of $x=2$	Right of $x=2$
$\frac{f(1)-f(0)}{1-0} = \frac{\frac{11}{-1}-(-4)}{1}$	$\frac{f(4)-f(3)}{4-3} = \frac{10-17}{1}$
$= -7$	$= -7$
decreasing	decreasing

v)



Graph not to scale

Just a rough sketch

36)

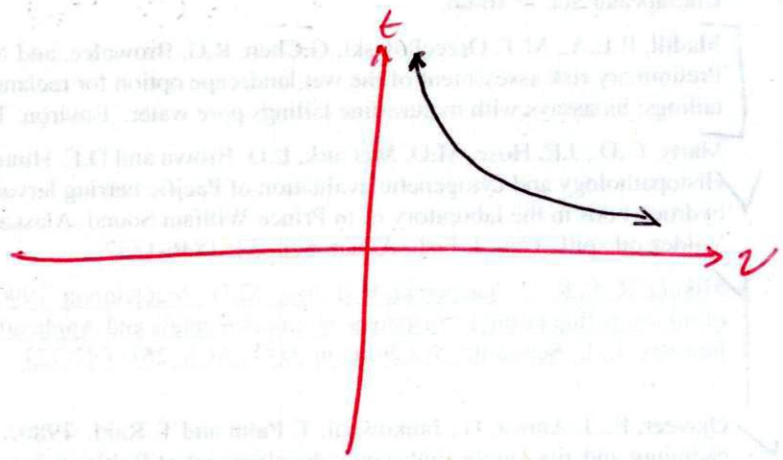
a) $\frac{5\text{km}}{t} = \vec{v}(t)$ $\quad t>0$, $v>0$

To find reciprocal make t a function of v:

$$\frac{5}{t(v)} = v$$

so $t(v) = \frac{5}{v}$ when $v \neq 0$

b) HA at $t=0$
and VA at $v=0$
No intercepts
t and $v > 0$
(no such thing as negative time)



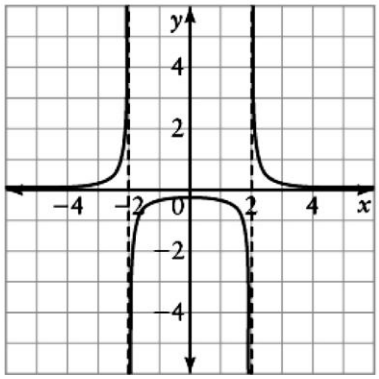
c) $v = 4.5 \frac{\text{km}}{\text{h}}$

so $t(4.5) = \frac{5\text{ km}}{4.5 \frac{\text{km}}{\text{h}}} = 1.11\text{h} \times \frac{60\text{ min}}{1\text{h}} = 66.6\text{ min}$ or 67 min.

∴ It takes Alistair 67 min to complete a 5 km run at $4.5 \frac{\text{km}}{\text{h}}$

d) As Alistair's speed increases, the rate of change of his run time decreases

37. Write an equation for the graph of the rational function shown. Explain your reasoning.



V.As at $-2, 2$ so $x \neq -2, 2$

HA at 0 so $y \neq 0$

Parabola between -2 and 2 $\quad y_{int} \approx$ at $-\frac{1}{4}$

↳ midpt of parabola is at $x=0$

$$f(x) = \frac{K}{(x+2)(x-2)}$$

$K > 0$. K can be any positive number because we don't know how stretched the graph is.

38. Determine an equation in the form $f(x) = \dfrac{ax+b}{cx+d}$ for a function that has asymptotes with equations $x = -1$ and $y = \dfrac{3}{4}$ and a *y*-intercept of 2. Sketch a graph of your function.

$f(x) = \frac{ax+b}{cx+d}$ with V.As at $x=-1$ and HAs at $y=\frac{3}{4}$ and y_{int} at $y=2$

So $x \neq -1$, $y \neq \frac{3}{4}$

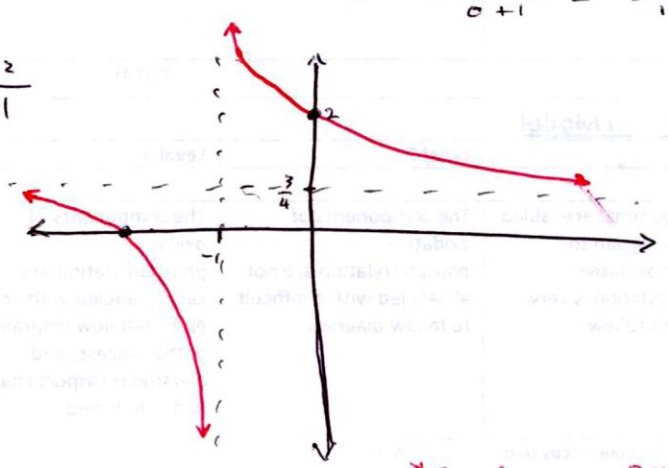
V.A at $x+1=0$, or $x=-1$ ✓

$f(x) = \dfrac{\frac{3}{4}x+2}{x+1}$

H.A. $\dfrac{\frac{3}{4}\frac{x}{x}+\frac{2}{x}}{\frac{x}{x}+\frac{1}{x}} = \dfrac{\frac{3}{4}+\frac{2}{x}}{1+\frac{1}{x}} \xrightarrow{\text{as } x\to\infty} \dfrac{\frac{3}{4}+0}{1+0} \to \dfrac{3}{4}$

y_{int} at $f(0) = \dfrac{\frac{3}{4}(0)+2}{0+1} = \dfrac{0+2}{1} = 2$ ✓

Thus $f(x) = \dfrac{\frac{3}{4}x+2}{x+1}$



left $f(-0.5) = \frac{13}{4} = 3.25$

$f(0) = 2$

right $f(1) = \frac{11}{8}$ decr.

left $f(-3) = \frac{1}{8}$

$f(-2) = -\frac{1}{2}$

$f(-1.5) = -\frac{7}{4}$

right decr.

x_{int}: $0 = \frac{3}{4}x+2$

$-2 = \frac{3}{4}x$

$x = -2 \times \frac{4}{3} = -\frac{8}{3}$

* $f(x) = \dfrac{3x+8}{4x+4}$ is also an acceptable answer