

MHF4U1-ASSIGNMENT CHAPTER 2

NAME: _____

K: /60

I: /28

C: /16

A: /8

Multiple Choice [K: 32 marks]

Identify the choice that best completes the statement or answers the question.

- C 1. If $x^3 - 4x^2 + 5x - 6$ is divided by $x - 1$, then the restriction on x is
 a. $x \neq -4$ $x-1 \neq 0$ c. $x \neq 1$
 b. $x \neq -1$ so $x \neq 1$ d. no restrictions
- C 2. What is the remainder when $x^4 + 2x^2 - 3x + 7$ is divided by $x + 2$?
 a. 25 $x^4 + 0x^3 + 2x^2 - 3x + 7$ c. 37
 b. 13 d. 9
- d 3. If $6x^4 - 2x^3 - 21x^2 + 7x + 8$ is divided by $3x - 1$ to give a quotient of $2x^3 - 7x$ and a remainder of 8, then which of the following is true?
 a. $\frac{6x^4 - 2x^3 - 21x^2 + 7x + 8}{3x - 1} = 2x^3 - 7x + \frac{8}{3x - 1}$
 b. $6x^4 - 2x^3 - 21x^2 + 7x + 8 = (3x - 1)(2x^3 - 7x) + 8$
 c. $x \neq \frac{1}{3}$
 d. all of the above
- d 4. When $P(x) = 4x^3 - 4x + 1$ is divided by $2x - 3$, the remainder is
 a. $2x^2 + 3x + \frac{5}{2}$ $4x^3 + 0x^2 - 4x + 1$ c. $P\left(-\frac{3}{2}\right) = -\frac{13}{2}$
 b. $P(3) = 97$ d. $P\left(\frac{3}{2}\right) = \frac{17}{2}$
- C 5. For a polynomial $P(x)$, if $P\left(-\frac{3}{5}\right) = 0$, then which of the following must be a factor of $P(x)$?
 a. $x - \frac{3}{5}$ $P\left(\frac{b}{a}\right)$ c. $5x + 3$
 b. $3x + 5$ so $b = -3$ $a = 5$ d. $5x - 3$
- a 6. Which of the following binomials is a factor of $x^3 - 6x^2 + 11x - 6$?
 a. $x - 1$ $a = 1, b = 1$ c. $x + 7$ $a = 1, b = -7$
 b. $x + 1$ $a = 1, b = -1$ d. $2x + 3$ $a = 2, b = -3$
 Play out all 4 scenarios to see which $P\left(\frac{b}{a}\right) = 0$
- b 7. Which set of values for x should be tested to determine the possible zeros of $x^3 - 2x^2 + 3x - 12$?
 a. 1, 2, 3, 4, 6, and 12 c. $\pm 1, \pm 2, \pm 3, \pm 4, \text{ and } \pm 6$
 b. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \text{ and } \pm 12$ d. $\pm 2, \pm 3, \pm 4, \pm 6, \text{ and } \pm 12$
- C 8. Which of the following is the fully factored form of $x^3 - 6x^2 - 6x - 7$?
 a. $(x - 7)(x + 1)^2$ c. $(x - 7)(x^2 + x + 1)$
 b. $(x - 7)(x + 1)(x - 1)$ d. $(x - 6)(x + 1)(x - 1)$
- Expand each* →

- C 9. Which of the following is the factored form of $x^4 - 2x^2 - 3$? $\rightarrow -1, 3 \text{ or } 3$
- $(x-1)(x+1)(x-3)$
 - $(x^2-1)(x+3)$
 - $(x^2+1)(x^2-3)$
 - $(x+1)(x-1)(x+3)(x-3)$

- C 10. One root of the equation $x^3 + 2x - 3x^2 - 6 = 0$ is
- Test each $P(\frac{b}{c})$ to find it = 0.
- $-3 \rightarrow (x+3) \text{ so } P(-\frac{3}{1})$
 - $-1 \rightarrow (x+1) \text{ so } P(-1)$
 - $3 \rightarrow (x-3) \text{ so } P(\frac{3}{1})$
 - $1 \rightarrow (x-1) \text{ so } P(1)$

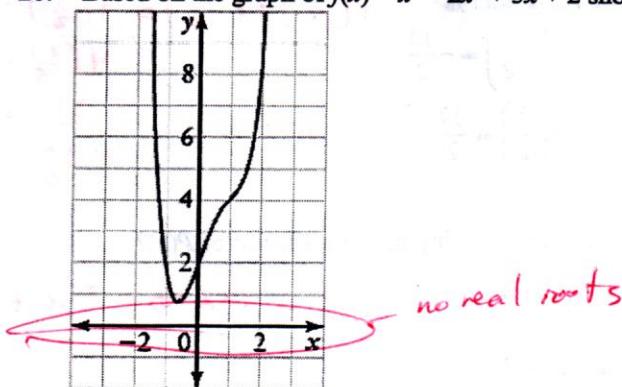
- b 11. What is the maximum number of real distinct roots that a quartic equation can have?
- infinitely many
 - 4
 - 2
 - none of the above
- degree 4

- b 12. Determine the value of k so that $x-3$ is a factor of $x^3 - 3x^2 + x + k$.
- $P(3) = 0 = (3)^3 - 3(3)^2 + (3) + k$
- $a=1 \ b=3 \ c=1 \ d=-1$
- $0 = 27 - 27 + 3 + k$
- $k = -3$
- $k = 3$
 - $k = -3$
 - $k = 1$
 - $k = -1$

- b 13. Which of the following is the fully factored form of $x^3 + 3x^2 - x - 3$?
- Expand each \rightarrow
- $a. (x+3)(x^2-1) = x^3 + 3x^2 - x - 3 \text{ BUT...}$
- $b. (x-1)(x+1)(x+3) \quad (x^2-1) \text{ factors further to } (x+1)(x-1)$
- $c. x^2(x+3) - (x+3)$
- $d. (x^2-1)(x-3)$

- b 14. If 2 is one root of the equation $4x^3 + kx - 24 = 0$, then the value of k is
- $P(\frac{2}{1}) = 0 = 4(2)^3 + k(2) - 24$
- $a. -1 \quad (x-2) \text{ so } a=1, b=2$
- $b. -4$
- $P(\frac{2}{1}) = 0$
- $c. 8$
- $d. \text{impossible to determine}$
- $0 = 4(8) + 2k - 24$
- $k = -4$

- c 20. Based on the graph of $f(x) = x^4 - 2x^3 + 3x + 2$ shown, what are the real roots of $x^4 - 2x^3 + 3x + 2 = 0$?



- 2
- 2, -1, 1, 2
- impossible to determine
- no real roots

- a 16. A family of quadratic functions has zeros -3 and 5 . Which of the following is a member of this family? degree 2
- $y = -\frac{1}{2}(x+3)(x-5)$
 - $y = 2(x+3)^2(x-5)$
 - $y = 4(x+3)(x-5)(x+15)$
 - $y = (x+5)(x-3)$

a 13. Find k if $2x+1$ is a factor of $kx^3 + 7x^2 + kx - 3$.

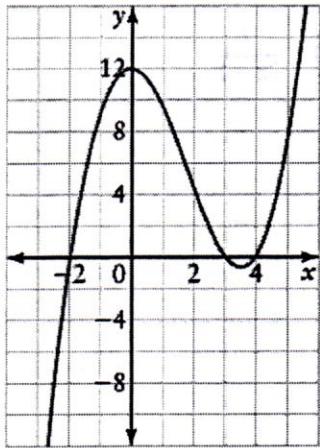
a. $k = -2$ b. $k = 2$ c. $k = \frac{11}{5}$ d. $k = 4/5$

$P(-\frac{1}{2}) = 0 = k(-\frac{1}{2})^3 + 7(-\frac{1}{2})^2 + k(-\frac{1}{2}) - 3$
 $= k(-\frac{1}{8}) + 7(\frac{1}{4}) + k(-\frac{1}{2}) - 3$
 $= k(-\frac{1}{8}) + k(-\frac{1}{2}) + \frac{7}{4} - 3$
 $0 = k(-\frac{1}{8} - \frac{1}{2}) - \frac{5}{4}$
 $\frac{5}{4} = k(-\frac{5}{8})$
 $\frac{5}{4} \times (-\frac{8}{5}) = k$
 $k = -2$

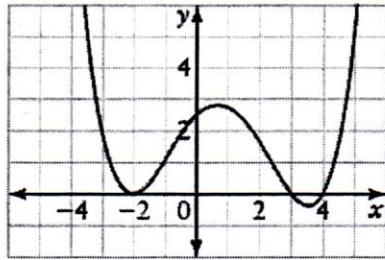
" x^3 rules out c and d"

- a 18. Which of the following graphs of polynomial functions corresponds to a cubic polynomial equation with roots $-2, 3$, and 4 ?

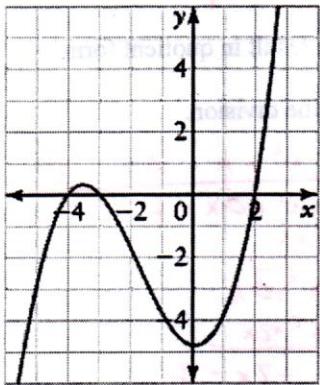
a.



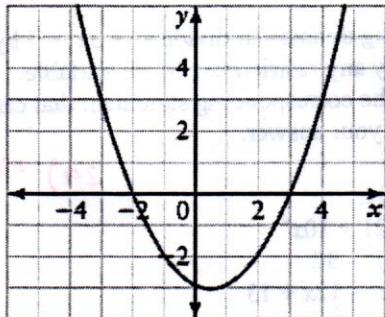
c.



b.



d.

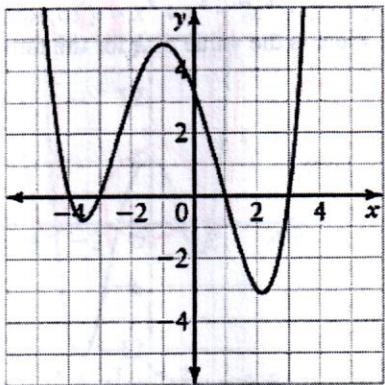


- c 19. Which of the following represents a family of cubic polynomials with zeros $-4, 2$, and 6 ?
- a. $y = (x+4)(x-2)(x-6)$
 b. $y = (x-4)(x+2)(x+6)$
 c. $y = k(x+4)(x-2)(x-6)$
 d. $y = k(x-4)(x+2)(x+6)$

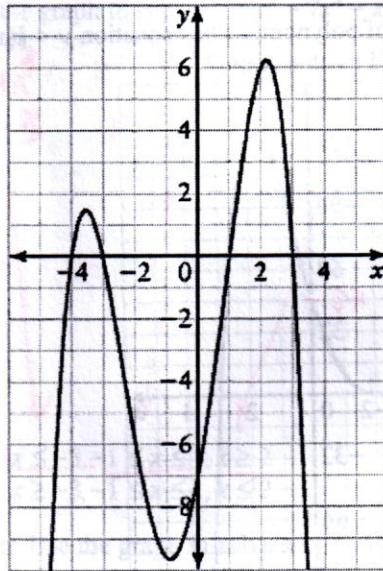
d

20. Examine the graphs of the following polynomial functions. Which graph does not belong to the same family?

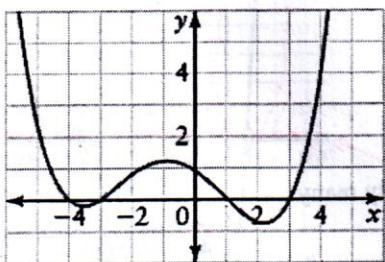
a.



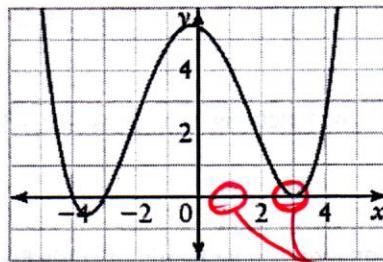
c.



b.



d.

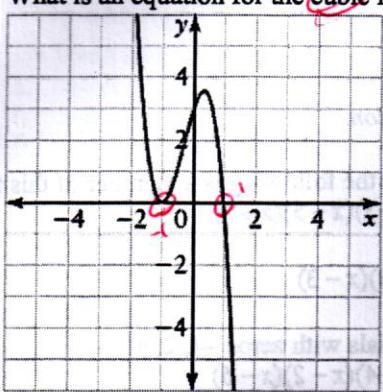


Different
x-intercepts

d

21. What is an equation for the cubic function represented by this graph?

degree 3

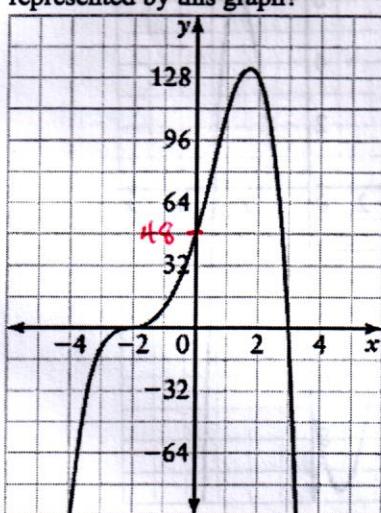


- x-intercepts are $-1, 1$
- and at $x = -1$ has order 2 because it doesn't cross axis.
- also Quadrant II to IV means negative leading coefficient

- a. $y = 3(x - 1)^2(x + 1)$
 b. $y = 3(x + 1)^2(x - 1)$

- c. $y = -3(x - 1)^2(x + 1)$
 d. $y = -3(x + 1)^2(x - 1)$

- d** 22. A family of polynomials has equation $y = k(x - 3)(x + 2)^3$. What is the value of k for the family member represented by this graph? er



at $x = 0$, $y = 48$

Plug it into your equation

$$48 = k(0 - 3)(0 + 2)^3$$

$$48 = k(-3)(2)^3$$

$$48 = k(-3)(8)$$

$$48 = k(-24)$$

$$k = -2$$

- a. $k = 48$
b. $k = -48$

- c. $k = 2$
d. $k = -2$

- d** 23. How many cubic functions have zeros -10 , -5 , and 4 ? you can make k-value anything

- a. 1
b. 2

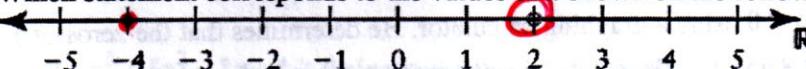
- c. 3
d. infinitely many

- C** 24. What is an equation for the family of cubic functions with zeros -8 , 3 , and $\frac{4}{3}$? cubics

- a. $y = (x + 8)(x - 3)(4x - 3)$
b. $y = (x + 8)(x - 3)(3x - 4)$

- c. $y = k(x + 8)(x - 3)(3x - 4)$
d. $y = k(x + 8)(x - 3)(4x - 3)$

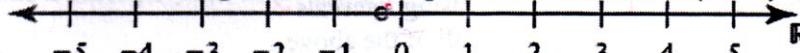
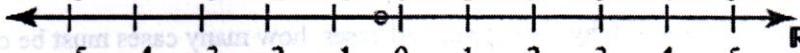
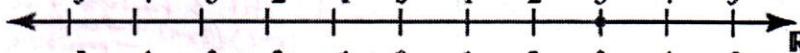
- d** 25. Which statement corresponds to the values of x shown on the following number line?



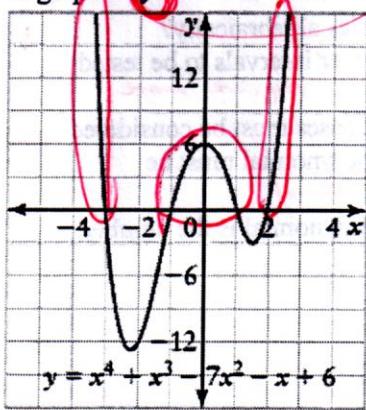
- a. $-4 \leq x < 2$
b. $-4 < x \leq 2$

- c. $-4 \leq x > 2$
d. $-4 \geq x > 2$

- a** 26. Which of the following number lines depicts the solution to $3x(x^2 - 4) + 2x > 3x^3 + 5(x + 1)$?

- a. 
b. 
c. 
d. 

- C 27. A graph of $y = x^4 + x^3 - 7x^2 - x + 6$ is shown. Use the graph to solve $x^4 + x^3 - 7x^2 - x + 6 \geq 0$.

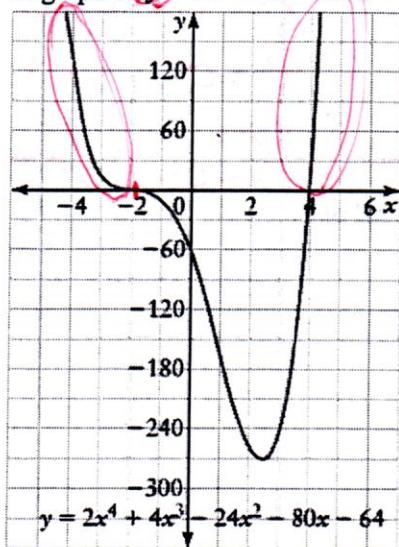


$$\begin{aligned} & -\infty \text{ to } -3, -1 \text{ to } 1, 2 \text{ to } \infty \\ \text{or} \quad & x \leq -3, -1 \leq x \leq 1, x \geq 2 \end{aligned}$$

- a. $-1 \leq x \leq 1$
b. $-3 \leq x \leq -1, 1 \leq x \leq 2$

- c. $x \leq -3, -1 \leq x \leq 1, x \geq 2$
d. $x \geq -3, -1 \leq x \leq 1, x \leq 2$

- b 28. A graph of $y = 2x^4 + 4x^3 - 24x^2 - 80x - 64$ is shown. Use the graph to solve $-2x^4 - 4x^3 + 24x^2 + 80x + 64 \geq 0$.



$$x \leq -2 \text{ and } x \geq 4$$

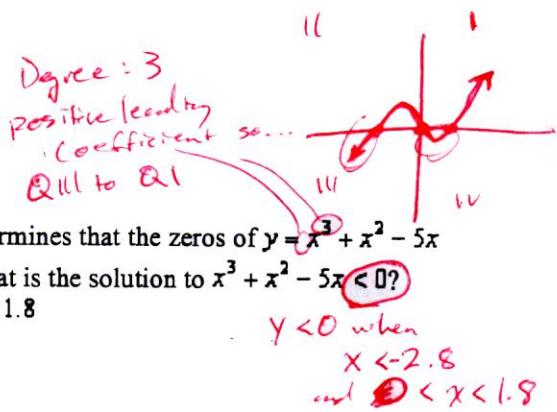
- a. $x = -2, 4$
b. $x \leq -2, x \geq 4$
c. $-2 \leq x \geq 4$
d. $-2 \leq x \leq 4$

- b 29. Toby is solving $x^3 + x^2 - 5x < 0$ using a graphing calculator. He determines that the zeros of $y = x^3 + x^2 - 5x$

are 0 and approximately -2.8 and 1.8 . Based on this information, what is the solution to $x^3 + x^2 - 5x < 0$?

- a. $x = 0, x = -2.8, x = 1.8$
b. $x < -2.8, 0 < x < 1.8$

- c. $-2.8 < x < 0, x > 1.8$
d. $-2.8 < x < 1.8$



d

30. The inequality $x^4 - 5x^2 + 4 \geq 0$ can be solved by

- a. considering all cases ✓ $\pm 1, \pm 2, \pm 4$
- b. graphing ✓
- c. using intervals ✓
- d. all of the above

C

31. When solving $3(x-2)(2x-1)(x+4) \geq 0$ by considering all cases, how many cases must be considered?

- a. 2 $x=2 \quad x=\frac{1}{2} \quad x=-4$
- b. 3 Case 1: $x \leq -4$
- c. 4
- d. 8

Case 2: $-4 \leq x \leq 0.5$

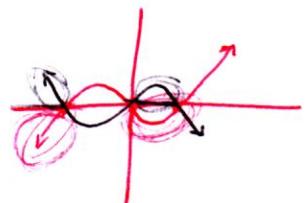
Case 3: $0.5 \leq x \leq 2$

Case 4: $x \geq 2$

d

32. Which inequality has $x \leq -4$ or $0 \leq x \leq 3$ as its solution?

- a. $x(x-3)(x+4) \leq 0$ → odd degree, +ve L.C.
- b. $-2x(x-3)(x+4) \geq 0$ → odd degree, -ve L.C.
- c. $100x(x-3)(x+4) \leq 0$ → odd degree, +ve L.C.
- d. all of the above



Notice that choices a, b, and c are all part of the same family since they all have the same 3 x-intercepts. You can draw a table to demonstrate all 4 cases to consider in the inequality to solve this question.

(If you don't know how, please ask Mr. Choi)

Short Answer

33. Find the solution(s) for x in $x(4x-3)(x+1) = 0$ [I: 3 marks]

34. Find the solution(s) to $x^2(x^2 + 1) > 0$ [I: 3 marks]

35. a) Use long division to divide $x^3 + 3x^2 - 7$ by $x + 2$. Express the result in quotient form. [K: 3 marks]

b) Identify any restrictions on the variable. [K: 1 mark]

c) Write the corresponding statement that can be used to check the division. [K: 1 mark]

d) Using what you wrote for part c), verify your answer. [K: 2 marks]

40. Solve the following inequalities using an algebraic method.

a) $x^2 + x - 2 > 0$ [I: 1 mark] [K: 3 marks]

b) $x(x-2)(x-1)(x+1) \leq 0$ [K: 5 marks]

c) $x^3 \geq 9x$ [I: 1 mark] [K: 3 marks]

d) $-x^3 + 2x^2 + 4x - 8 \geq 0$ [I: 2 marks] [K: 4 marks]

33. Find the solution(s) for x in $x(4x - 3)(x + 1) = 0$ [I: 3 marks]

$$x(4x - 3)(x + 1) = 0$$

Set each factor
to zero

$$x = 0$$

$$(4x - 3) = 0$$

$$(x + 1) = 0$$

Then solve for each x

$$x = 0$$

$$x = 3/4$$

$$x = -1$$

34. Find the solution(s) to $x^2(x^2 + 1) > 0$ [I: 3 marks]

Look at each factor in the equation

$$x^2 \text{ and } (x^2 + 1)$$

No matter what value
you choose for x
both factors stay positive

For $x^2 > 0$ x simply must not equal 0

so...

$$\{x | x \in \mathbb{R}, x \neq 0\}$$

35) a)

$$\begin{array}{r} x^2 + x - 2 \\ \hline x+2 \overline{)x^3 + 3x^2 + 0x - 7} & R \\ - x^3 - 2x^2 \\ \hline x^2 + 2x \\ - x^2 - 2x \\ \hline - 2x - 7 \\ - - 2x - 4 \\ \hline - 3 \end{array}$$

$$\frac{x^3 + 3x^2 - 7}{x+2} = x^2 + x - 2 + \frac{-3}{x+2}$$

b) $x \neq -2$

same

c) $\frac{x^3 + 3x^2 - 7}{x+2} = x^2 + x - 2 + \frac{-3}{x+2}$

d) $x^3 + 3x^2 - 7 = (x+2)\left(x^2 + x - 2 - \frac{3}{x+2}\right)$

$$= x^2(x+2) + x(x+2) - 2(x+2) - \frac{3}{x+2}(x+2)$$

$$= x^3 + 2x^2 + x^2 + 2x - 2x - 4 - 3$$

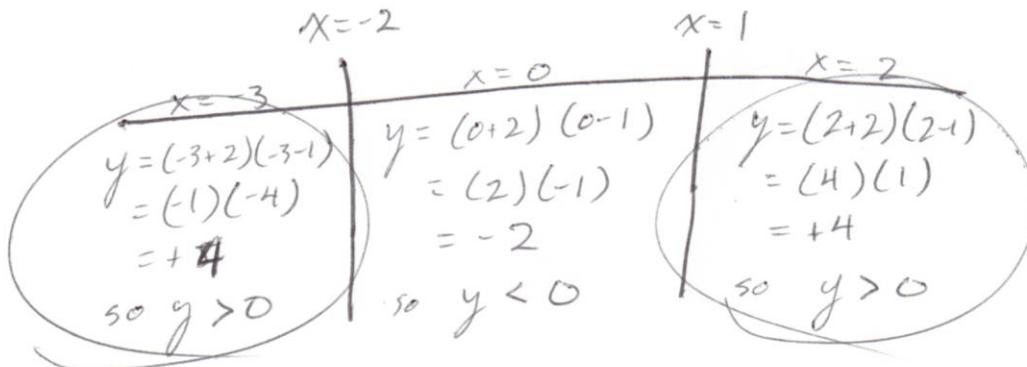
$$= x^3 + 3x^2 - 7$$

40)

a) $x^2 + x - 2 > 0$

$$(x+2)(x-1) > 0$$

x -intercepts: $x = -2$ $x = 1$

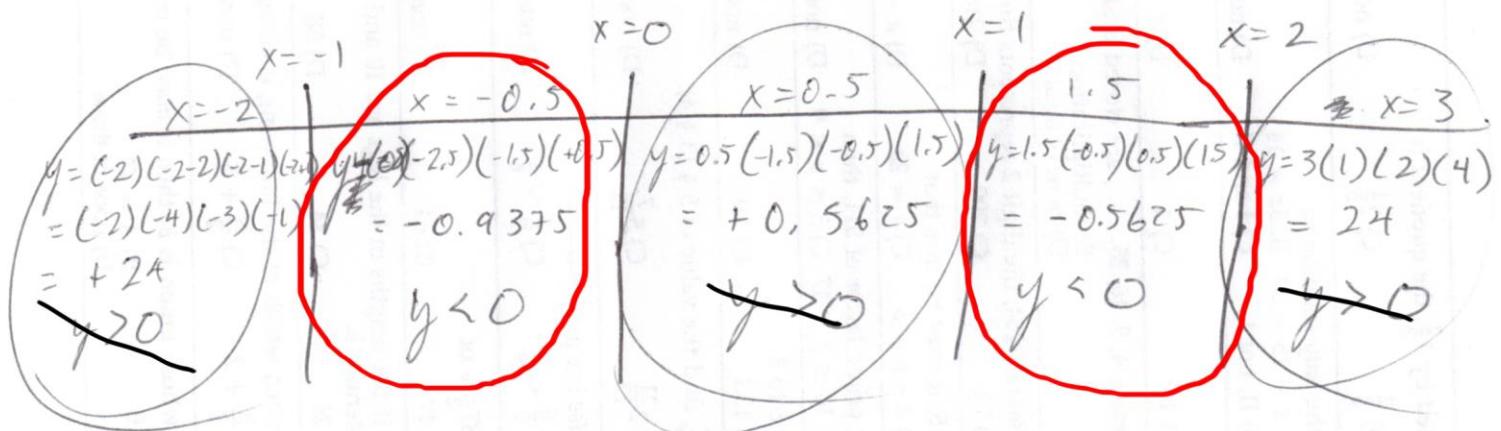


$\therefore (x+2)(x-1) > 0 \text{ when } x < -2 \text{ and } x > 1$

b)

$$x(x-2)(x-1)(x+1) \leq 0$$

$x = 0$ $x = 2$ $x = 1$ $x = -1$



so $x(x-2)(x-1)(x+1) \leq 0$

when $-1 \leq x \leq 0, 1 \leq x \leq 2$

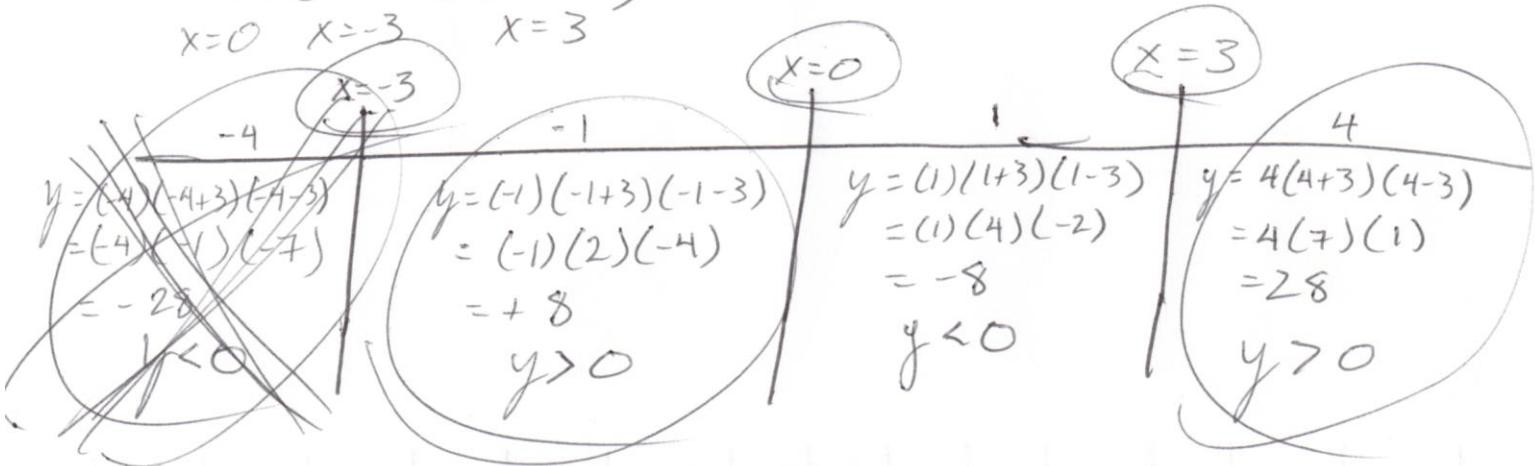
$$c) x^3 \geq 9x$$

$$x^3 - 9x \geq 0$$

$$x(x^2 - 9) \geq 0$$

$$x(x+3)(x-3) \geq 0$$

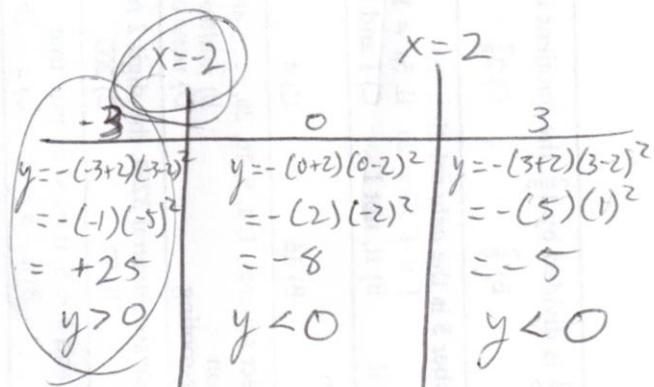
$$x=0 \quad x=-3 \quad x=3$$



so $x(x+3)(x-3) \geq 0$ when

$-3 \leq x \leq 0$ and when $x \geq 3$

$$\begin{aligned}
 d) \quad & -x^3 + 2x^2 + 4x - 8 \geq 0 \\
 & -(x^3 - 2x^2 - 4x + 8) \geq 0 \\
 & -(x^2(x-2) - 4(x-2)) \geq 0 \\
 & -(x-2)(x^2-4) \geq 0 \\
 & -(x-2)(x+2)(x-2) \geq 0 \\
 & -(x+2)(x-2)^2 \geq 0
 \end{aligned}$$



so $-(x+2)(x-2)^2 \geq 0$

when $x \leq -2$

$$41) P(x) = 6x^3 + mx^2 + nx - 5$$

$(x+1)$ is a factor
so $\frac{b}{a} = -1$

$(x-1)$ has a remainder of -4 , so $\frac{b}{a} = 1$

$$P(-1) = 6(-1)^3 + m(-1)^2 + n(-1) - 5$$

$$0 = 6(-1) + m(+1) - n - 5$$

$$0 = -6 + m - n - 5$$

$$0 = -11 + m - n$$

$$m = n + 11.$$

$$P(1) = 6(1)^3 + m(1)^2 + n(1) - 5$$

$$-4 = 6 + m + n - 5$$

$$-4 = 1 + m + n$$

$$0 = 1 + 4 + m + n$$

$$0 = 5 + m + n$$

$$m = -n - 5$$

Make them equal

$$n + 11 = -n - 5$$

$$2n = -5 - 11$$

$$2n = -16$$

$$n = -8$$

$$m = -(-8) - 5$$

$$m = 8 - 5$$

$$m = 3$$

so $m = 3, n = -8$

$$42) x+t \text{ so } \frac{b}{a} = -t$$

$$P(-t) = (-t+t)^4 + (-t+c)^4 - (t-c)^4$$

$$= (0)^4 + (-t+c)^4 - (t-c)^4$$

$$= ((-1)(t-c))^4 - (t-c)^4$$

$$= (-1)^4 (t-c)^4 - (t-c)^4$$

$$= (+1) (t-c)^4 - (t-c)^4$$

$$= 0$$

$$43) -2 \text{ is a root of } x^3 + x = -4x^2 + 6$$

so $x+2$ is a factor

$$x^3 + x + 4x^2 - 6 = 0$$

$$x^3 + 4x^2 + x - 6 = 0$$

$$\begin{array}{r} x^2 + 2x - 3 \\ \hline x+2 \overline{) x^3 + 4x^2 + x - 6} \\ - x^3 - 2x^2 \\ \hline 2x^2 + x \\ - 2x^2 - 4x \\ \hline - 3x - 6 \\ - - 3x - 6 \\ \hline 0 \end{array}$$

Thus $x^3 + 4x^2 + x - 6 = (x+2)(x^2 + 2x - 3)$

$$= (x+2)(x+3)(x-1)$$

so we have $x = -2$, $x = -3$, and $x = 1$
as our roots.

$$44) V = l \times w \times h$$

$$= l \times l \times h$$

$$= l \times l(l+4)$$

but we have a square base
so $l=w$

We also know height is 4cm more than the l .

We also know $V = 225 \text{ cm}^3$

$$225 = l^2(l+4)$$

$$225 = l^3 + 4l^2$$

$$0 = l^3 + 4l^2 - 225$$

Test factors of 225

$$\frac{b}{a} = \pm 1, \pm 5, \pm 9, \pm 15, \pm 25, \pm 45, \pm 225$$

$$P(5) = 5^3 + 4(5)^2 - 225$$

$$= 125 + 100 - 225$$

$$= 0 \quad \therefore (x-5) \text{ is a factor}$$

~~This~~ This means $l=5$ and $h=5+4=9$

\therefore The dimensions are 5cm by 5cm by 9cm.

45)

$$V = l \times w \times h$$
 ~~$V = 1 \times 2 \times 4 = 8$~~

Amit needs to increase each dimension by x amount, so that the volume is $9x$ the original

so

 ~~$\boxed{\text{Diagram}}$~~

$$9 \times 8 = (x+1)(x+2)(x+4)$$

$$72 = (x^2 + x + 2x + 2)(x + 4)$$

$$72 = (x^2 + 3x + 2)(x + 4)$$

$$72 = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8$$

$$0 = x^3 + 7x^2 + 14x + 8 - 72$$

$$0 = x^3 + 7x^2 + 14x - 64$$

Factors of 64

$$\text{for } \frac{b}{a} = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$$

$$P(2) = 2^3 + 7(2)^2 + 14(2) - 64$$

$$= 8 + 28 + 28 - 64$$

$$= 0 \quad \therefore x = 2 \text{ is a solution.}$$

\therefore Amit should increase his dimensions by 2 m.

$$46) -x^3 + 5x^2 - 8x + 4 \geq 0$$

$$-(x^3 - 5x^2 + 8x - 4) \geq 0$$

Possible factors

$$\text{for } \frac{b}{a} = \pm 1, \pm 2, \pm 4$$

$$P(1) = -(1)^3 - 5(1)^2 + 8(1) - 4$$

$$= -(1 - 5 + 8 - 4)$$

$$= 0$$

$\therefore x=1$ is a solution
and $(x-1)$ is a factor

$$\begin{array}{r} x^2 - 4x + 4 \\ x-1 \sqrt{x^3 - 5x^2 + 8x - 4} \\ \underline{-x^3 - x^2} \\ -4x^2 + 8x \\ \underline{-4x^2 + 4x} \\ 4x - 4 \\ \underline{4x - 4} \\ 0 \end{array}$$

$$\text{so } -(x^3 - 5x^2 + 8x - 4) \geq 0$$

$$\text{becomes } -(x-1)(x^2 - 4x + 4) \geq 0$$

$$-(x-1)(x-2)^2 \geq 0$$

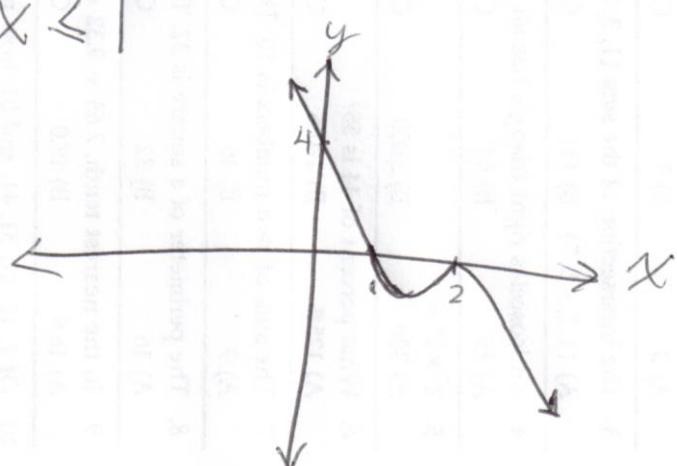
$$x=1 \quad x=2$$

when $x \leq 1$

$$x_{\text{int}} = 1, 2$$

$$y_{\text{int}} = +4$$

$x=0$	$x=1$	$x=1.5$	$x=2$	$x=2.5$
$y = -(0-1)(0-2)^2$	$y = -(1-1)(1-2)^2$	$y = -(1.5-1)(1.5-2)^2$	$y = -(2.5-1)(2.5-2)^2$	
$= -(-1)(-2)^2$	$= -(0.5)(-0.5)^2$	$= -(1.5)(0.5)^2$	$= -(1.5)(0.5)^2$	
$= +4$	$= -0.125$	$= -0.375$	$= -0.375$	
$y > 0$	$y < 0$	$y < 0$	$y < 0$	



$$47) 3x^2(x^2-8)+6x+5 < 4x^4-6x(4x-1)+4$$

$$3x^4 - 24x^2 + 6x + 5 < 4x^4 - 24x^2 + 6x + 4$$

$$\cancel{3x^4} - \cancel{4x^4} - \cancel{24x^2} + \cancel{24x^2} + \cancel{6x} - \cancel{6x} + 5 - 4 < 0$$

$$-x^4 + 1 < 0$$

$$-(x^4 - 1) < 0$$

$$-(x^2+1)(x^2-1) < 0$$

$$-(x^2+1)(x+1)(x-1) < 0$$

$x = -1$ $x = 1$

