MHF4U1 ASSIGNMENT CHAPTER 1 **NAME:**_____

K: /39 **I:** /35 **A:** /6

Multiple Choice [K: 1 mark each = 33 total marks]
Identify the choice that best completes the statement or answers the question.

A 1. An equation representing a function that extends from quadrant 3 to quadrant 1 is
   a. $y = x^3$ c. $y = 2x^6$
   b. $y = -2x^5$ d. $y = -5x^4$

odd and positive L.C.

D 2. An equation representing a function that extends from quadrant 3 to quadrant 4 is
   a. $y = x^3 + 7x - 1$ c. $y = 2x^6 - 4x^3$
   b. $y = -2x^5 + x - 1$ d. $y = -5x^4 - 2x^2 - 1$

Even, negative L.C.

A 3. The degree of the polynomial function $y = x^3 - 2x^2 + 5x - 1$ is
   a. 3 c. 5
   b. 4 d. 6

D 4. The graph of the polynomial function $y = -2x(x-1)^2(x-2)^2$ extends from
   a. quadrant 3 to quadrant 1 c. quadrant 2 to quadrant 1
   b. quadrant 3 to quadrant 4 d. quadrant 2 to quadrant 4

$-2x^5$ — odd w/ negative L.C.

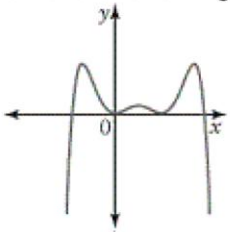
C 5. The function $y = 6(x-1)^4(x-2)^2(x+1)$ changes sign at
   a. $x = 1$ c. $x = -1$
   b. $x = 2$ d. it doesn't change sign

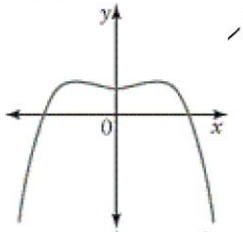
$(x-1)^4$ has order 4
$(x-2)^2$ has order 2
$(x+1)$ has order 1

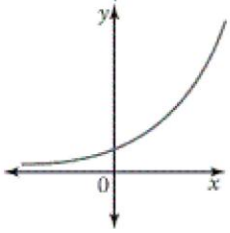
D 6. Which of the following *is* a polynomial function?
   a. $y = \sin x$ c. $y = 3^x$
   b. $y = \cos x$ d. $y = x^3$

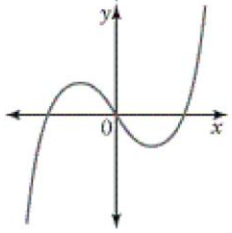
* Polynomial functions changes sign at roots that are of ODD order but not that are of EVEN order *

B 7. Which of the following is an even function?
   a. $y = 2x^4 + x^3$ c. $y = 2x^4 - x$
   b. $y = 2x^4 + 11$ d. $y = -x^3 + x^5$

C 8. Which of the following graphs represents an even function?
   a.


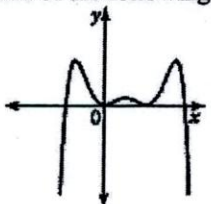
   c.


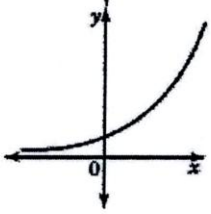
   b.


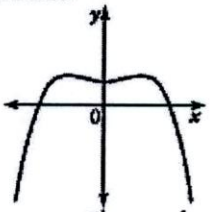
   d.


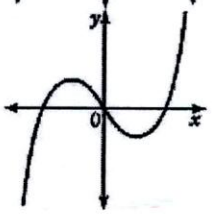
- symmetrical abt the y-axis
* Can't be (a) because NOT symmetrical about the y-axis.

D 9 Which of the following graphs represents an odd function?

a. 

b. 

c. 

d. 

Odd function
↳ symmetry about the origin.

C 10 The number of times that the function $y = (x-1)^3(x+2)(x-4)^2$ changes sign is

a. 0
b. 1
c. 2
d. 3

orders are 3, 1, 2.
Changes sign at ODD orders thus sign changes twice, because there's 2 odd exponents

D 11 The function $y = (x-4)^2(x-7)(x+3)^3$ is negative on the intervals

even $x=4$ odd $x=7$ odd $x=-3$

a. $x \in (-\infty, -3)$ and $x \in (4, 7)$
b. $x \in (-\infty, 3)$ and $x \in (7, \infty)$
c. $x \in (-3, 4)$ and $x \in (7, \infty)$
d. $x \in (-3, 4)$ and $x \in (4, 7)$

positive L.C. and degree of 6 (even) so quadrants II to I.

x int:	-3	4	7
order:	odd	even	even

B 12 The table of values represents a polynomial function.

		1st Δ	Δ 2nd
–3	6	~~X~~	~~X~~
–2	2	-4	X
–1	0	-2	2
0	0	0	2
1	2	2	2
2	6	4	2

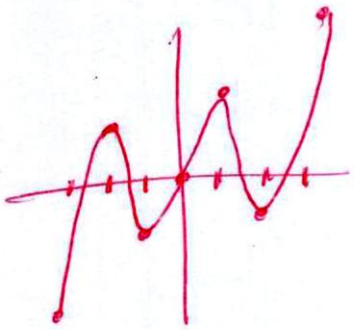
Degree: 2

The function is

a. linear
b. quadratic
c. cubic
d. quartic

D 13. The table of values represents a polynomial function.

–3	–7
–2	2
–1	–3
0	0
1	3
2	–2
3	7

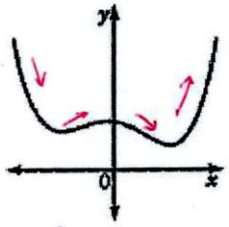


Turn it upside down and it'll look the same

The function appears to be

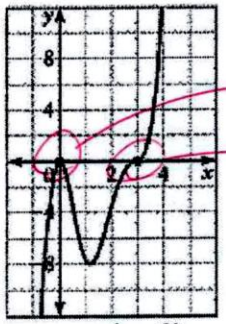
a. not symmetric
b. symmetric about the x-axis
c. symmetric about the y-axis
d. symmetric about the origin

C 14. The least possible degree of the polynomial function represented by the graph shown is



a. 2
b. 3
c. 4
d. 5

D 15. An equation for the graph shown is



Doesn't change sign ∴ even order ie) x^2

Flattens out ∴ higher than one order ie) $(x-3)^3$

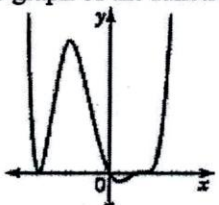
a. $y = x(x-3)$
b. $y = x(x-3)^3$
c. $y = x^2(x-3)$
d. $y = x^2(x-3)^3$

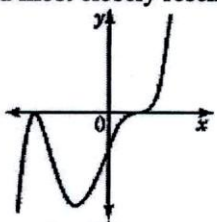
x^6 positive L.C. means Quadrants 2 to 1. even degree

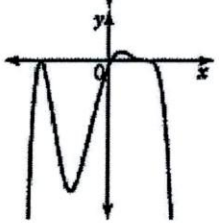
A 16. The graph of the function $y = x(x-1)^3(x+2)^2$ would most closely resemble

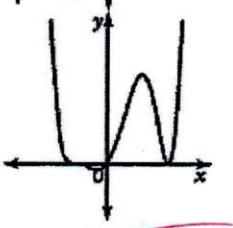
even order ∴ no sign change at $x = -2$

high odd order means flattens out at $x = 1$

a. 

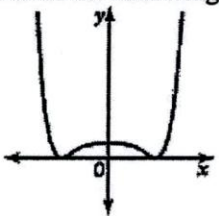
c. 

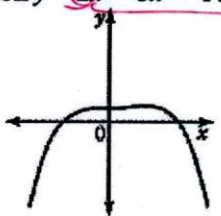
b. 

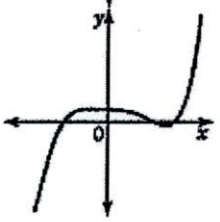
d. 

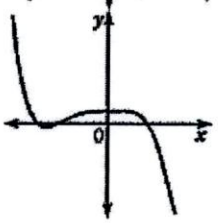
B 17. Which of the following graphs represents the function $y = 2x^5 - 3x^4 + 1$?

odd, positive L.C. } Quadrants III to I

a. 

c. 

b. 

d. 

D 18. Given the function $y = (x-1)^2(x+1)^2$, which finite differences will be equal (or constant)?

x^4 Degree 4

a. first differences
b. second differences
c. third differences
d. fourth differences

D 19. Given the function $y = -3x^2 - 5x + 1$, the second differences will all equal

$\Delta = a n!$
$\Delta = -3(2!)$
$= -6$

a. 3
b. –3
c. 6
d. –6

C 20. An equation for a cubic function with zeros 1, –2, and 3 that passes through the point (2, 12) is

$a(x-1)(x+2)(x-3)$

a. $y = x(x+2)(x-3)$
b. $y = (x-1)(x+2)(x-3)$
c. $y = -3(x-1)(x+2)(x-3)$
d. $y = \frac{1}{2}(x+1)(x-2)(x+3)$

$12 = a(2-1)(2+2)(2-3)$
$12 = a(1)(4)(-1)$ → $\frac{12}{-4} = a = -3$ ∴ $y = -3(x-1)(x+2)(x-3)$

B 21. An equation for a quintic function with zeros 1, 0, and 2 that passes through the point (–1, 24) is

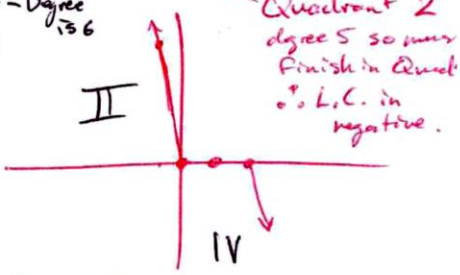
degree = 5

a. $y = 2x(x-1)(x+2)^3$
b. $y = -2x^2(x-1)^2(x-2)$
c. $y = -3(x-1)^2(x-2)^2x^2$ Degree is 6
d. $y = \frac{1}{2}x^3(x-1)(x-2)$

has positive L.C.

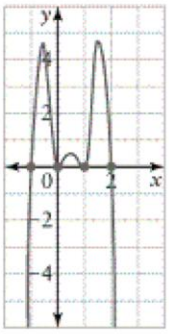
has positive L.C.

Quadrant 2 degree 5 so must finish in Quad IV ∴ L.C. is negative.



MHF4U1 ASSIGNMENT CHAPTER 1 **NAME:**_____

A 22. State the interval(s) for which the graph of the function is negative.



a. $x \in (-\infty, -1)$ and $x \in (2, \infty)$
b. $x \in (-1, 2)$
c. $x \in (-1, 0)$ and $x \in (2, \infty)$
d. $x \in (-1, 0)$ and $x \in (0, 2)$

a 23. Given a function of the form $y = a[k(x - d)]^n + c$, where $k > 0$, the transformation that occurs by changing the value of k is
a. a horizontal stretch or compression
b. a vertical stretch or compression
c. a vertical translation
d. a reflection in the x-axis

b 24. The graph of the function $y = x^4$ is transformed to the graph of the function $y = -2(x - 3)^4 + 1$ by
a. a horizontal stretch by a factor of 2, a reflection in the x-axis, a translation of 3 units to the left, and a translation of 1 unit up
b. a vertical stretch by factor of 2, a reflection in the x-axis, a translation of 3 units to the right, and a translation of 1 unit up
c. a vertical stretch by a factor of 2, a reflection in the x-axis, a translation of 3 units to the left, and a translation of 1 unit up
d. a vertical compression by a factor of $\frac{1}{2}$, a reflection in the x-axis, a translation of 3 units to the left, and a translation of 1 unit up

$a = -2$
$d = +3$
$c = +1$

D 25. If the graph of the function $y = x^3$ is compressed horizontally by a factor of $\frac{1}{2}$, stretched vertically by a factor of 3, and translated 5 units to the left, an equation for the graph of the transformed function is
a. $y = 3\left[\frac{1}{2}(x + 5)\right]^3$
b. $y = 3[2(x - 5)]^3$
c. $y = 6(x + 5)^3$
d. $y = 24(x + 5)^3$

$a = 3$
$k = 2$
$d = -5$

$y = 3[2(x+5)]^3$
$= 3[2^3(x+5)^3]$
$= 3(8)[(x+5)^3]$
$= 24[(x+5)]^3$

A

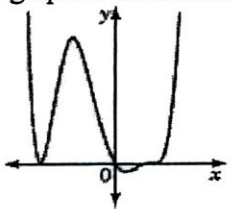
The graph of the function $y = x(x-1)^3(x+2)^2$ would most closely resemble

$x_{int} = 0, 1, -2$

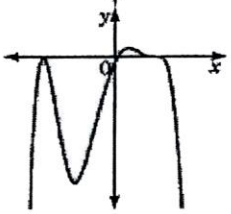
no change in sign at $x = -2$

Degree: 6 +ve L.C. means Quadrants 2 to 1.

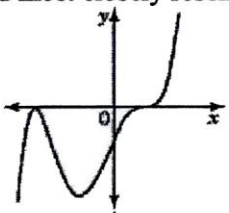
a.



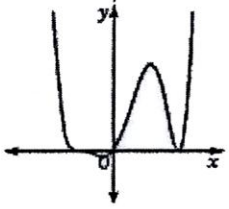
b.



c.



d.

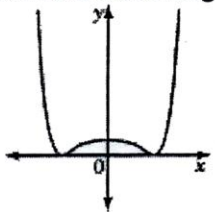


Degree 5 with +ve L.C. means Quads 3 to 1

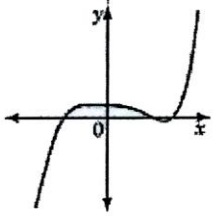
b

27. Which of the following graphs represents the function $y = 2x^5 - 3x^4 + 1$?

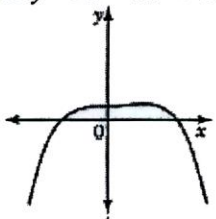
a.



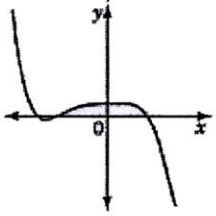
b.



c.

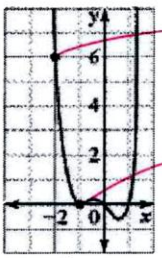


d.



d

28. A secant drawn through the points shown on the graph has a slope of



(-2, 6)

(-1, 0)

so... $\frac{\Delta y}{\Delta x} = \frac{0-6}{-1-(-2)} = \frac{-6}{-1+2} = \frac{-6}{1} = -6$

a. –1
b. –2
c. –3
d. –6

$y(-1) = (-1)^2 - (-1) - 1 = 1$

$y(4) = (4)^2 - (4) - 1 = 11$

b

29. The average rate of change of the function $y = x^2 - x - 1$ from $x = -1$ to $x = 4$ is

a. 1
b. 2
c. 2.2
d. 11

$\frac{\Delta y}{\Delta x} = \frac{11-1}{4-(-1)} = \frac{10}{5} = 2$

b

30. The number of people, P, at a playground after t min is given by $P = t^3 + 4t + 20$.
The average rate of change of the number of people at the playground from 3 min to 4 min is

a. 39 people/min
b. 41 people/min
c. 59 people/min
d. 100 people/min

$P(3) = 59$

$P(4) = 100$

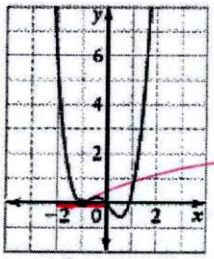
$\frac{100-59}{4-3} = \frac{41}{1} = 41$

C 31. The number of people, P, at a playground after t min is given by $P = t^3 + 4t + 20$..
The instantaneous rate of change of the number of people at the playground after 1 min is approximately

a. 5 people/min
b. 6 people/min
c. 7 people/min
d. 25 people/min

$P(1) = 25$
$P(1.1) = 25.731$
$\frac{25.731 - 25}{1.1 - 1} = \frac{0.731}{0.1}$
$= 7.31 \approx 7 \frac{\text{people}}{\text{min}}$

C 32. A tangent to the graph of the function at the point shown has a slope of

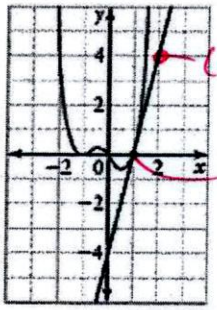


tangent located at local min

flat slope
∴ slope is 0,

a. –2
b. –1
c. 0
d. 1

d 33. The slope of the tangent at the point indicated on the graph is



Take 2 points on line
$\frac{\Delta y}{\Delta x} = \frac{4 - 0}{2 - 1}$
$= \frac{4}{1}$
$= 4$

a. $\frac{1}{2}$
b. 1
c. 2
d. 4

Completion
Complete each statement.

34. The polynomial function $y = x(x - 1)(x + 2)^2$ has 3 x-intercepts.

35. The polynomial function $y = 2x^9(x^2 - 1)$ is an example of a(n) odd function.

36. The graph of the function $y = x^4(x - 1)^6(x + 2)^2$ changes sign 0 times.

37. For the polynomial function $y = x^5 - 3x^4 - x + 1$, the fifth differences will be constant (equal).

Short Answer

38. Determine the type of polynomial function (linear, quadratic, cubic, etc.) that the table of values represents.

x	y	1st	2nd
–3	34	X	X
–2	17	-17	X
–1	6	-11	6
0	1	-5	6
1	2	1	6
2	9	7	6
3	22	13	6

Degree = 2

∴ Quadratic

39. The table of values represents a polynomial function. Determine the value of the constant finite differences.

x	y	1st	2nd	3rd	4th
–3	169	X	X	X	X
–2	35	-134	X	X	X
–1	3	-32	102	X	X
0	1	-2	30	-72	X
1	5	4	6	-24	48
2	39	34	30	24	48
–3	175	136	102	72	48

$\Delta = a(n!)$

$48 = a(4!)$

$48 = a(24)$

$a = 2$

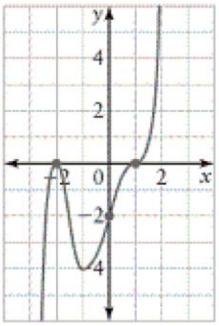
40. Determine an equation for a cubic polynomial function with zeros 1, 2, and 3.

$P(x) = (x-1)(x-2)(x-3)$

41. Determine an equation for a polynomial function with zeros 0 (order 2), 5 (order 2), and $\frac{1}{2}$

$P(x) = x^2(x-5)^2\left(x-\frac{1}{2}\right)$

42. Determine an equation for the graph of the polynomial function shown. [I: 4 marks]



$x_{int} = -2, +1$ $\quad y_{int} = -2$

Domain	$x<-2$	$-2<x<1$	$x>1$
$f(x)$	-ive	-ive	+ive

sign doesn't change $\therefore x_{int}: -2$ is of order 2

* Also at $x=1$, graph flattens out $\therefore$ order is 3

So with $y(0) = -2$

$-2 = a(x+2)^2(x-1)^3$

$-2 = a(2)^2(-1)^3$

$-2 = a\,4(-1)$

$-\frac{2}{-4} = a = \frac{1}{2}$ $\quad \therefore y = \frac{1}{2}(x+2)^2(x-1)^3$

Problem

43. Determine an equation for the polynomial function represented in the table of values. [I: 5 marks]

x	y	1st	2nd
–3	0	X	X
–2	–4	-4	X
–1	–6	-2	2
0	–6	0	2
1	–4	2	2
2	0	4	2
3	6	6	2

degree = 2

$2 = a(2!)$

$2 = a(2)$

$a = 1$

Also $x_{int}: -3, 2$

$y_{int}: -6$

$\therefore y = (x+3)(x-2)$

44. Determine an equation for the quartic polynomial function represented by the table of values. [I: 5 marks]

x	y	1st	2nd	3rd	4th
–3	91	X	X	X	X
–2	21	-70	X	X	X
–1	3	-18	52	X	X
0	1	-2	16	-36	X
1	3	2	4	-12	24
2	21	18	16	12	24
3	91	70	52	36	24

$24 = a(4!)$

$24 = a(24)$

$a = 1$

Symmetrical abt y-axis $\therefore$ it's an even function: $y = ax^4 + bx^2 + c$

$y_{int}: (0,1) \leftarrow$ vertex

so $a = 1$ and $c = 1$

Plug in any pt from table like $(-1, 3)$

$3 = (-1)^4 + b(-1)^2 + 1$

$3 = 1 + b + 1$

$b = 1$ $\quad \therefore y = x^4 + x^2 + 1$

45. Determine an equation in factored form for a polynomial function with zeros –1 (order 2) and 3 (order 3) that passes through the point (4, 5) [I: 3 marks]

$P(x) = a(x+1)^2(x-3)^3$

$5 = a(4+1)^2(4-3)^3$

$5 = a(5)^2(1)^3$

$5 = a\,25$

$a = \frac{1}{5}$ $\quad \therefore P(x) = \frac{1}{5}(x+1)^2(x-3)^3$

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46. Determine the slope of the secant on the graph of the function $y = x^2$ from $x = 0$ to $x = 1$. [K: 2 marks]

$\frac{\Delta y}{\Delta x} = \frac{1-0}{1-0} = 1$ $m = 1$

47. Determine the average rate of change of the function $y = 2x^4 - x^2$ from $x = -2$ to $x = 2$. [K: 2 marks]

$y(-2) = 28$
$y(2) = 28$
$\frac{28-28}{2-(-2)} = \frac{0}{4} = 0$ $m = 0$

48. Estimate the slope of the tangent to the graph of the function $y = 2x^3 + x^2 + 23$ at $x = 2$. [K: 2 marks]

$y(2) = 43$
$y(2.1) = 45.932$
$\frac{45.932-43}{2.1-2} = \frac{2.932}{0.1}$
$= 29.32$

$m \approx 29.32$

49. The number of toy kangaroos, K, in a toy box after t days is given by $K = t^2 + 20t$.
Estimate the instantaneous rate at which the number of kangaroos is changing after 3 days. [A: 3 marks]

$y(3) = 69$

Choose an x-value very close to 3

$y(3.1) = 71.61$

$m = \frac{\Delta y}{\Delta x} = \frac{71.61-69}{3.1-3} = \frac{2.61}{0.1} = 26.1$

$\approx 26 \frac{\text{kangaroos}}{\text{day}}$

18. The amount of money, M, in dollars, in a piggy bank after t days is given by $M = 2t^3 + 2t + 10$.
Estimate the instantaneous rate of change of the amount of money in the piggy bank after 2 days. [A: 3 marks]

$M(2) = 30$ $M(2.1) = 32.722$

$M = \frac{\Delta y}{\Delta x} = \frac{32.722-30}{2.1-2} = \frac{2.722}{0.1} = 27.22$

$\approx \$27$/day is inserted in the piggy bank.